The Model of Demand for Substitute Durable Goods

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Introduction

Durable goods are production equipment or machinery which life is usually over three years [3, s. 537]. These goods are represented often by certain furnishing of the household of the decided type and homogenous character. There are included items such as TV sets, refrigerators, automobiles or computers.

It may seem that the durable goods form a special and narrow problem. However, the problem involves other questions. One of them is solution of services which accompany consumption in a long-term prospective.

Next question relates with goods production. When new goods are being introduced to the market, it is necessary to reply in a certain manner to satiation of consumption. Another reason is at the moment prevailing marketing orientation of firms at a customer. Different types of fidelity systems become matters of course and firms share the opinion that keeping the customer is as important as his acquiring.

Essentially, the model approaches of durable goods problem abstract from „momentary" influences and concentrate upon long-term trend in time. Models describe the gradual satiation of consumption in the long-term view [5, s. 291 – 292].

The present models analyze behavior of demand for durable goods individually. In this article we concentrate on competitive environment of this type of goods. Our aim is to construct a model which describes behavior of demand for substitute, i. e. mutually competitive, durable goods at a certain market.

Construction of the Model

Let us consider a given market which consists of $n \geq 2$ substitute durable goods. The main goal of this article is to describe how demand for the given n durable goods will change depending on time. We use the mathematical theory from field of differential equations systems in our modelling.

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Let us denote by function $x_i = x_i(t)$ demand for the i-th goods, for $i = 1, 2, \ldots, n$, $n \geq 2$. Under the demand for i-th goods at the time t we shall understand the size of the market of i-th goods at the time t. The size of the market of i-th goods at the time t represents the quantity of sold units of the given goods to the time t.

Price, quality and advertising belong among the most important factors influencing demand for goods.

The most significant influence determining how big is demand for goods is their price. Generally, when the price of goods grows (and other factors do not change), demand decreases and vice versa. Lower prices attract new buyers and the existing ones increase their shopping. We shall denote the coefficients of the unit price of i-th goods at the time t by the function $p_i(t)$.

Another key factor influencing demand for substitute goods is their quality. Customers obviously at the same price prefer goods of higher quality. We shall express the coefficients of the unit quality of goods $1, 2, \ldots, n$ by the functions $q_i(t)$, $1, 2, \ldots, n$.

Let the constant S represent how big is the given market (so called capacity of the market). The capacity of the market can be measured by possible maximum quantity of sold units of all goods on the market. Market demand depends on how well the customers are informed about the given goods what is implemented by the firms by means of advertising.

We shall denote the influence of advertising (which can be approximated by coefficients of advertising costs of the firms for their goods) by the functions $a_i(t)$, $\mathrm{i} = 1, 2, \ldots, n$.

We shall formulate the general model in the form

$$\begin{aligned} x_1' &= f_1(x_1, x_2, \ldots, x_n) \\ x_2' &= f_2(x_1, x_2, \ldots, x_n) \\ &\vdots \\ x_n' &= f_n(x_1, x_2, \ldots, x_n) \end{aligned} \tag{1}$$

where the following requirements are put on the functions f_i:

(i) Relationships of the competition negatively influence all the substitute goods. With growing demand for one goods of them, decreases (in boundary cases does not change) the rate of the demand growth for the competitive goods. This means that the function f_i is not growing in variables x_1, x_2, $\ldots, x_{i-1}$, $x_{i+1}, \ldots, x_n$:

$$\frac{\partial f_i}{\partial x_1} \le 0,\ \frac{\partial f_i}{\partial x_2} \le 0,\ \ldots,\ \frac{\partial f_i}{\partial x_{i-1}} \le 0,\ \frac{\partial f_i}{\partial x_{i+1}} \le 0,\ \ldots,\ \frac{\partial f_i}{\partial x_n} \le 0$$

$\forall\ x_i$ defined, $i = 1,\ 2, \ldots, n$.

(ii) The durable goods create the market where the level of satiation (capacity) is S. Thus points $(S,\ 0, \ldots, 0)$, $(0,\ S,\ 0, \ldots, 0)$, ..., $(0, \ldots, 0,\ S)$ represent equilibrium solution of the system (1):

$$f_1(S,\ 0, \ldots, 0) = f_2(0,\ S,\ 0, \ldots, 0) = \ldots = f_n(0, \ldots, 0,\ S) = 0.$$

(iii) The functions f_i are non-negative. This condition represents gradual satiating of demand which is typical right for the durable goods.

The requirements (i) – (iii) are met by the model in the following form

$$\begin{aligned} \frac{dx_1}{dt} &= \left(S - \sum_{i=1}^{n} x_i\right)\left(g_1(t)x_1 + h_1(t)\right) \\ \frac{dx_2}{dt} &= \left(S - \sum_{i=1}^{n} x_i\right)\left(g_2(t)x_2 + h_2(t)\right) \\ &\vdots \\ \frac{dx_n}{dt} &= \left(S - \sum_{i=1}^{n} x_i\right)\left(g_n(t)x_n + h_n(t)\right) \end{aligned} \qquad (2)$$

where $x_i \ge 0$, $i = 1,\ 2, \ldots, n$, $\Sigma_{i=1}^{n} x_i \le S$ and $g_i(t)\, x_i + h_i(t) \ge 0$, $\forall\ x_i$, $x_i \ge 0$, $i = 1,\ 2, \ldots, n$, $\Sigma_{i=1}^{n} x_i \le S$. The function $x_i = x_i(t)$ represents demand for i-th goods and the constant S denotes the capacity of the given market.

The expression $S - \Sigma_{i=1}^{n} x_i$ represents gradual satiating of the market, thus decreasing of the rate of demand size for individual goods. It depends on size of the market S.

The terms $g_i(t)x_i + h_i(t)$, $i = 1,\ 2, \ldots, n$, relate to respective goods. Based on the given factors which influence demand for goods, we can specify more precisely the functions $g_i(t)$, $h_i(t)$. The functions $g_i(t)$ will include the coefficients of the unit prices $p_i(t)$ and the coefficients of unit quality $q_i(t)$ of individual goods. The functions $h_i(t)$ take into account the influence of advertising $a_i(t)$.

In the simplest case the functions $p_i(t)$, $g_i(t)$, $a_i(t)$, $i = 1,\ 2, \ldots, n$ are constant in the model (2). Demand for goods grows with increasing quality and volume of advertising. On the contrary, high prices of goods slow down growth of demand. So let us put $g_i(t) = q_i - p_i$, $h_i(t) = a_i$, $\forall i = 1, 2, \ldots, n$, where $q_i \neq p_i$ $\forall i = 1, 2, \ldots, n$. The cases $q_i = p_i$ are very unlikely and we shall not consider them.

After such a modification we obtain the nonlinear system of n differential equations with constant coefficients in the form

$$\begin{aligned}
\frac{dx_1}{dt} &= \left(S - \sum_{i=1}^{n} x_i\right)\left((q_1 - p_1)x_1 + a_1\right) \\
\frac{dx_2}{dt} &= \left(S - \sum_{i=1}^{n} x_i\right)\left((q_2 - p_2)x_2 + a_2\right) \\
&\vdots \\
\frac{dx_n}{dt} &= \left(S - \sum_{i=1}^{n} x_i\right)\left((q_n - p_n)x_n + a_n\right)
\end{aligned} \tag{3}$$

where q_i, p_i, a_i, S are positive constants, $q_i \neq p_i$, $i = 1, 2, \ldots, n$, we consider the system on the set

$$D = \left\{(x_1, x_2, \ldots, x_n) \in R^n : 0 \leq x_1 \leq S,\ 0 \leq x_2 \leq S - x_1,\ \ldots,\ 0 \leq x_n \leq S - \sum_{i=1}^{n-1} x_i\right\}$$

and $(q_i - p_i)x_i + a_i \geq 0$ $\forall x_1, x_2, \ldots, x_n,$ $0 \leq x_1, x_2, \ldots, x_n,$ $\sum_{i=1}^{n} x_i \leq S$. The functions $x_i = x_i(t)$, $i = 1, 2, \ldots, n$ denote demand for goods $1, 2, \ldots, n$ at the time t. The constants q_i represent the coefficients of unit quality of individual goods, p_i are coefficients of unit prices of goods, the constants a_i approximate advertising costs of individual firms on their goods and S is the capacity of the market.

All the solutions of the system (3) are nondecreasing functions. It implies that each solution is either equilibrium or converges as t approaches infinity towards the equilibrium solution.

Let us find all the equilibrium solutions $(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$ of the given system. The system (3) is in the equilibrium state if it holds

$$\begin{aligned}
0 &= \left(S - \sum_{i=1}^{n} \bar{x}_i\right)\left((q_1 - p_1)\bar{x}_1 + a_1\right) \\
0 &= \left(S - \sum_{i=1}^{n} \bar{x}_i\right)\left((q_2 - p_2)\bar{x}_2 + a_2\right) \\
&\vdots \\
0 &= \left(S - \sum_{i=1}^{n} \bar{x}_i\right)\left((q_n - p_n)\bar{x}_n + a_n\right)
\end{aligned} \tag{4}$$

The system (4) is met on the set D if $S-\sum_{i=1}^{n}\bar{x}_i=0$. Thus the equilibrium solutions are all the points $(\bar{x}_1,\bar{x}_2,\dots,\bar{x}_n)$ which satisfy the equation $S-\sum_{i=1}^{n}\bar{x}_i=0$, $\bar{x}_i\geq 0$, $i=1,\ 2,\dots,n$.

Example

Let us consider a given market consisting of two substitute durable goods. We denote demand of the goods 1 by function $x_1=x_1(t)$ and demand of the goods 2 by function $x_2=x_2(t)$. The following initial values of model (3) were chosen:

$$S=1000 \qquad q_1=0{,}0012\ q_2=0{,}0020$$
$$p_1=0{,}0010\ p_2=0{,}0015$$
$$a_1=0{,}0016\ a_2=0{,}0018$$

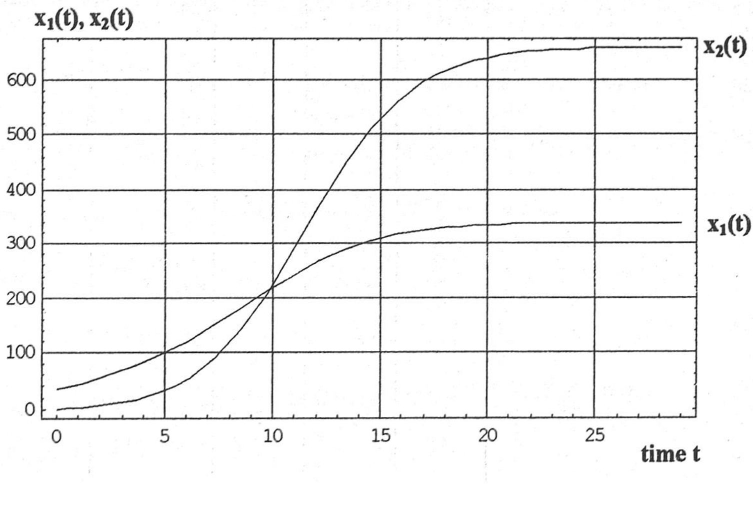
Substituting these values into model (3) we obtain

$$\frac{dx_1}{dt}=\left(1000-x_1-x_2\right)\left((0{,}0012-0{,}0010)\,x_1+0{,}0016\right)$$
$$\frac{dx_2}{dt}=\left(1000-x_1-x_2\right)\left((0{,}0020-0{,}0015)\,x_2+0{,}0018\right).$$

This system has infinite number of equilibrium solutions in the form $(\bar{x}_1,\bar{x}_2)=(\bar{x}_1,\ 1000-\bar{x}_1)$, where $\bar{x}_1\in\langle 0,\ 1000\rangle$.

Figure 1

Demand $x_1(t)$ and $x_2(t)$ in dependence on time t



The solution of the given system for initial conditions $x_1(0)=35$, $x_2(0)=0$ is shown in Figure 1. These conditions can be interpreted as follows. Our model starts at time $t=0$, when goods 1 has market size of 35 units and goods 2 is just entering the market.

In Figure 1 it is visible that in spite of unfavourable starting position of goods 2, approximately at the time $t=10$ both substitute goods are equal in market size. From this moment the second goods is starting to dominate in the market.

Further in Figure 1 we can see the equilibrium solution $(\bar{x}_1, \bar{x}_2)=(340, 660)$. The line $\bar{x}_1=340$ is asymptote to the graph of $x_1(t)$, the line $\bar{x}_2=660$ is asymptote to the graph of $x_2(t)$. The goods demand converges to this equilibrium values. The sum $\bar{x}_1+\bar{x}_2=340+660=1000$ which is equal to market capacity (S). Thus the market tends to its satiation. Under present assumptions about price, quality, advertising and market sizes of both goods at the time $t=0$, the first goods will obtain (after sufficiently long time) 34 per cent of the market and competitive goods remaining 66 per cent.

Conclusion

In the article we constructed a nonlinear system of differential equations, which models demand for substitute durable goods.

The main model (3) forms nonlinear system of n differential equations with constant coefficients (so called autonomous system). The model takes into account the factors as price, quality, advertising and market capacity. The output of the model is a vector function of demand size dependent on time and parameters of that model.

Each solution of the model is either equilibrium or converges as t approaches infinity towards the equilibrium solution. However, it means that the solution will not reach the equilibrium point at the finite time.

We found out that all the equilibriums of the model form a hyperplane where the sum of market size of all the substitute goods is equal to the level of the market satiation. Those equilibrium points represent states of competitive environment where market sizes of competitive goods do not change at time.

The model predicts a certain market share of the goods in terms of the given competitive environment. This market share is completely defined by firm price strategy, volume of advertising and quality level of their goods.

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MODEL DOPYTU PO SUBSTITUČNÝCH STATKOCH DLHODOBEJ SPOTREBY

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V článku sme skonštruovali model, ktorý opisuje priebeh dopytu po substitučných, t. j. navzájom si konkurujúcich, statkoch dlhodobej spotreby na istom trhu. Problematika statkov dlhodobej spotreby je rozšírená hlavne v nadväznosti na služby, ktoré túto spotrebu sprevádzajú. Hlavnou črtou dopytu po statkoch dlhodobej spotreby je skutočnosť, že dopyt tvorí neklesajúcu funkciu času. Opodstatnenosť neklesajúceho dopytu potvrdzujú aj rôzne typy vernostných systémov, ktoré podniky používajú, a ďalšie aktivity podnikov na udržiavanie si stálej klientely zákazníkov.

Hlavný model tvorí nelineárny systém n diferenciálnych rovníc s konštantnými koeficientmi. Model zohľadňuje faktor ceny, kvality, reklamy a kapacitu trhu. Výstupom modelu je vektorová funkcia veľkosti dopytu závislá od času a od zvolených parametrov.

Keďže funkcie dopytu sú neklesajúce, každé riešenie systému je buď rovnovážne, alebo konverguje pre $t \to \infty$ k rovnovážnemu riešeniu. Rovnovážne riešenie predstavuje stav konkurenčného prostredia, v ktorom sa veľkosti trhov substitučných tovarov v čase nemenia. Všetky rovnovážne riešenia modelu tvoria nadrovinu, na ktorej súčet veľkostí trhov všetkých konkurenčných tovarov je rovný hladine nasýtenia trhu.

Navrhnutý model umožňuje posúdiť vývoj dopytu po jednotlivých substitučných statkoch dlhodobej spotreby v danom konkurenčnom prostredí. Model predpovedá, aký trhový podiel v rámci konkurenčného prostredia si jednotlivé tovary vymedzia. Tento trhový podiel je úplne určený cenovou stratégiou podnikov, objemom reklamy a kvalitatívnou úrovňou tovarov.