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On Foreign Capital Investment Bifurcations[1]

*Miloslav S. VOSVRDA**

An economic model of cycles focused on the foreign capital investment phenomenon will be briefly introduced. We consider a system of the first order non-linear differential equations where a feedback is controlled by a capital/output ratio parameter. The influence of the parameter changes on a macroeconomic stability is analyzed. A convergence of this economic system either to stable state, or limit cycle, or chaotic state is demonstrated.

Introduction

There are two basic tools to analyze fundamental issues in dynamic macroeconomics. One of them is a model of optimal growth describing savings behavior. The second one is the Solow-Swan model with a constant aggregate propensity to save out of income. A steady state of the dynamical economic system corresponds to a growth path satisfying a stationary solution of a properly defined differential system and thus exhibiting certain conditions of constant growth rate in a single-sector model. One of the possible mechanisms for regulating the growth path is a savings rate. Regulation through the savings rate is made a possible by distinguishing two types of income, two social classes or by introducing money and financial assets (Henin, 1986). The standard neoclassical growth theory, Kaldor (1956), Pasinetti (1962), Samuelson and Modigliani (1966) were investigating the question to what extent different saving behavior of the two income groups (labor and capital) might influence the growth path. The case that each agent is able to save by accumulating capital but not to borrow from the other is solved by Bewley (1986). Woodford (1990) went beyond Bewley's results both showing that equilibrium cycles are possible and exhibiting conditions under which equilibrium dynamics are chaotic. The aggregate savings function needs no longer be concave, so that multiple and unstable steady states can occur. The role of differential

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simple savings behavior and distribution effects for stability of stationary states was investigated in Böhm and Kaas (2000). They demonstrated that the economy exhibits unstable steady states and fluctuations if the income distribution varies sufficiently and if shareholders save more than workers.

A central question of this paper is demonstrating how foreign capital investments change qualitative properties of the growth path of the economic system. The analysis will be provided by analyzing of the stability conditions of the economic system with a foreign financing and by the Hopf bifurcation analysis.

1. An Economic System with a Foreign Financing

We will analyze of the macroeconomic system where *Y (t)* is determined by the level of the investments financed by both the (domestic) savings and the foreign capital inflows. Specifications on our investigated system appearing the following form of equations are motivated by Bouali (1999). Our macroeconomic system is represented by the following system of the differential equations

$$\dot{S}(t) = a \cdot Y(t) + p \cdot S(t) \cdot \left(YP - Y^2(t)\right)$$
$$\dot{Y}(t) = v^{-1} \cdot \left(S(t) + F(t)\right)$$
$$\dot{F} \cdot (t) = m \cdot S(t) - r \cdot Y(t) \tag{1}$$

where

S – savings of (domestic) households,
Y – Gross Domestic Product,
YP – the potential GDP,
a – the variation of the marginal propensity to savings,
p – the ratio of capitalized profit,
v – the capital/output ratio,
F – the foreign capital inflow,
m – the capital inflow/savings ratio,
r – the debt refund/output ratio.

It is assumed that $\dfrac{a \cdot v}{r} > 1$ and $a > r$.

If *Y (t)* < *YP*, this case represents a possibility of remarkable increase of profits, the economic activities are not constrained. High profits are derived from these sectors their markets are not yet saturated. A quality of the capitalization of the profits, re-injecting its into the financing circuit, is expressed by a non-linearity in the equation (1). If *Y (t)* > *YP*, this case represents the possibility of inflation appearing. A lack of new investment opportunities modifies use of savings. The saving deviation is accelerated by introducing non-linearity between the *Y (t)* and *YP*.

1.1. A Stationary Equilibrium

This system possesses a unique equilibrium as follows

$$(S\ Y\ F)^T \tag{2}$$

where

$$Y = \sqrt{\frac{a \cdot m}{p \cdot r} + YP} \qquad S = \frac{r}{m} \cdot \sqrt{\frac{a \cdot m}{p \cdot r} + YP} \qquad F = -\frac{r}{m} \cdot \sqrt{\frac{a \cdot m}{p \cdot r} + YP}$$

For parameters $a = 0.05$, p = 0.8, $v = 1.25$, $m = 0.1$, $r = 0.04$, and for initials of savings of households $S\ (0) = 0.45$, of output $Y\ (0) = 0.8$, of a foreign investment $F\ (0) = 0.035$, and of the potential output $YP = 1$ we obtain the following solution

$$(0.43\ \ 1.07\ \ -0.43)^T$$

1.2. Necessary and Sufficient Stability Conditions

The Routh-Hurwitz theorem (Gandolfo, 1997) is used for expressing of the necessary and sufficient stability conditions. The Jacobian matrix of our system equation is

$$J = \begin{Vmatrix} p \cdot (YP - Y^2) & a - 2 \cdot p \cdot S \cdot Y & 0 \\ 1/v & 0 & 1/v \\ m & -r & 0 \end{Vmatrix} = \begin{Vmatrix} -\frac{a \cdot m}{r} & -a - 2 \cdot \frac{p \cdot r}{m} & 0 \\ 1/v & 0 & 1/v \\ m & -r & 0 \end{Vmatrix}_{\substack{Y = \sqrt{\frac{a \cdot m}{p \cdot r} + YP} \\ S = \frac{r}{m} \cdot \sqrt{\frac{a \cdot m}{p \cdot r} + YP} \\ F = -\frac{r}{m} \cdot \sqrt{\frac{a \cdot m}{p \cdot r} + YP}}} \tag{3}$$

with the characteristic equation

$$\chi(\lambda) = \lambda^3 + g \cdot \lambda^2 + h \cdot \lambda + q \tag{4}$$

where

$$g = -tr(J) = \frac{a \cdot m}{r} \qquad q = -|J| = 2 \cdot \frac{m}{v} \cdot \left(a + \frac{p \cdot r}{m}\right) \qquad h = \frac{r - (a - 2 \cdot p \cdot S \cdot Y)}{v} = a + \frac{r}{v} \cdot \left(1 + 2 \cdot \frac{p}{m}\right)$$

The coefficient h represents the principal minors of the Jacobian J. We solve the characteristic equation and we get the following eigenvalues. The equation

$$\lambda^3 + g \cdot \lambda^2 + h \cdot \lambda + q = 0$$

has one real and two complex conjugate roots if the discriminant

$\Delta = \left(\frac{g^3}{27} - \frac{g \cdot h}{6} + \frac{q}{2}\right)^2 + \left(\frac{h}{3} - \frac{g^2}{9}\right)^3$ is positive (Lorenz, 1997, p. 86). The estimating of a sign of the discriminant is, a little bit cumbersome work, therefore introduced in the appendix A. It is shown that the sign of this discriminant is, for the system (3), positive. In numerical form our case with the following values of the parameters $a = 0.05$, $p = 0.8$, $v = 1.25$, $m = 0.1$, $r = 0.04$ has the values of the variables as follows $g = 0.125$, $q = 0.025$, $h = 0.582$, and the value of the $\Delta = 0.0074$. By computing we really get one real and two complex conjugate roots $\lambda_0 = -0.102$, $\lambda_1 = -0.012 + 0.073i$, $\lambda_2 = -0.012 - 0.073i$. The Routh-Hurwitz theorem asserts that necessary and sufficient conditions for all roots of the equation (5) to have negative real parts are given by the following inequalities (Gandolfo, 1997, p. 222). The leading principal minors of the matrix J

$$M1 = g \quad M2 = \begin{Vmatrix} g & q \\ 1 & h \end{Vmatrix} \quad M3 = \begin{Vmatrix} g & q & 0 \\ 1 & h & 0 \\ 0 & g & q \end{Vmatrix}$$

must be positive, i. e., that

$$M1 = \frac{a \cdot m}{r} > 0 \qquad M2 = \left[a \cdot m \cdot \left(\frac{a}{r} - \frac{1}{v}\right) + 2 \cdot \frac{p}{v} \cdot (a - r)\right] > 0$$

$$M3 = 2 \cdot (a \cdot m + p \cdot r)\left[a \cdot m \cdot \left(\frac{a \cdot v}{r} - 1\right) + 2 \cdot p \cdot (a - r)\right] > 0$$

From these inequalities follows that

$$0 < r < a$$

$$0 < \frac{m}{2} < p < 1$$

In our numerical case we get $|M1| = 0.125$, $|M2| = 0.014$, and $|M3| = 0.0008$. Because these values of parameters are positive the necessary and sufficient stability conditions are fulfilled.

In the following examples a behavior of such stable economic system under changes parameters p, a, r, m respectively is demonstrated. The parameters v, the capital/output ratio, is considered as a constant in the analyzed period.

Example 1

Changing of the parameter p

Let us consider the following values of parameters $a = 0.3$, $v = 1.25$, $p = 0.8$, $m = 0.8$, $r = 0.2$, $YP = 1$, and we solve the system of differential equations with the following initials $Y(0) = (0.45 \;\; 0.8 \;\; 0.35)^T$.

The solution in the graphical form is shown in the following figure 1. The parameter p will be changed from the value 0.01 until 0.385 and next from 0.51 till 0.885 with the other parameters unchanged. For low values of this parameter p this system is in an unstable state. For increasing values of the parameter p this system converges in the stable state. Therefore this system under changing of the parameter p has the so-called sub critical Hopf bifurcation. It means that when the parameter p is increasing from $p = 0.01$ till 0.885, the single fixed point changes its stability, which is monitored in figures 2, 3. We will now change piecemeal values of the parameters p in the following way $p = 0.01, 0.385, 0.51, 0.885$.

Figure 1

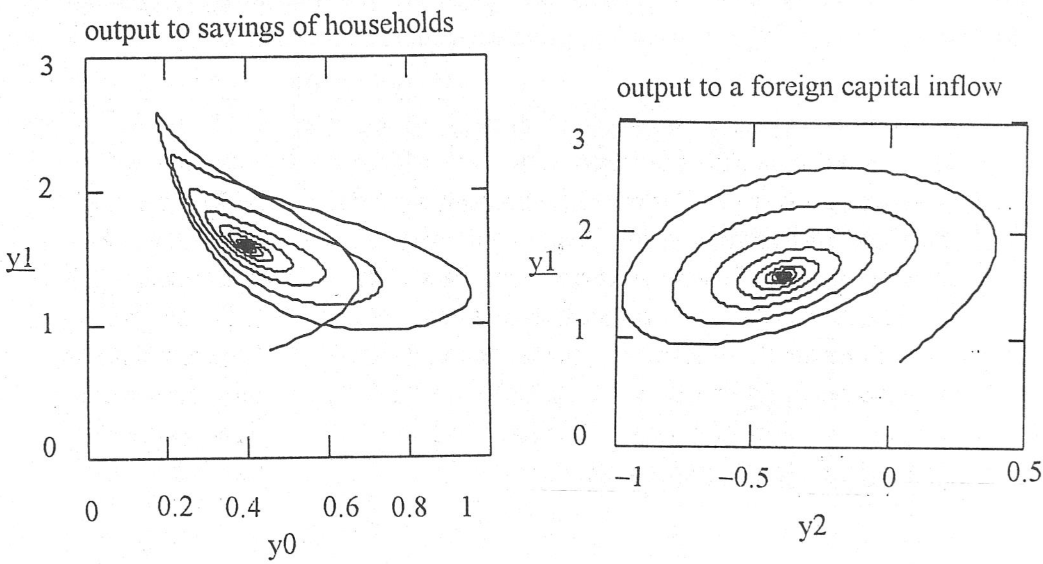


Figure 2

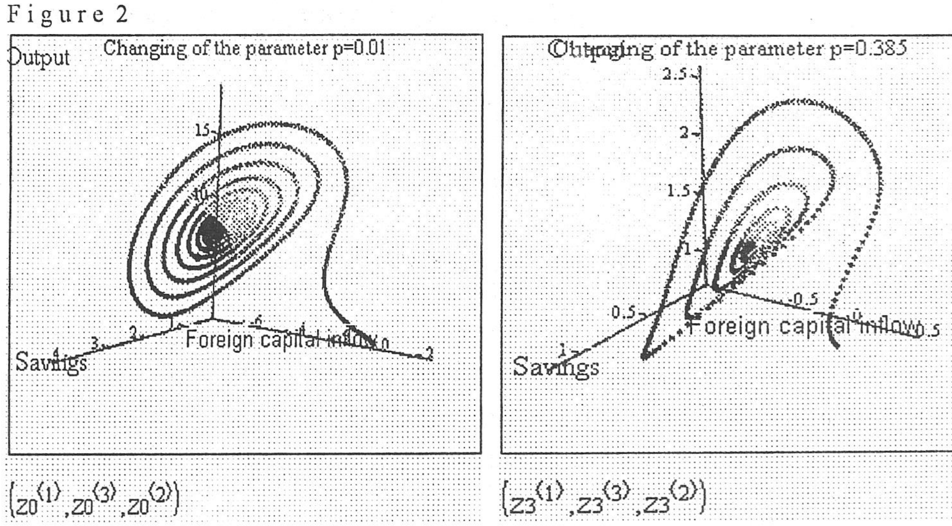
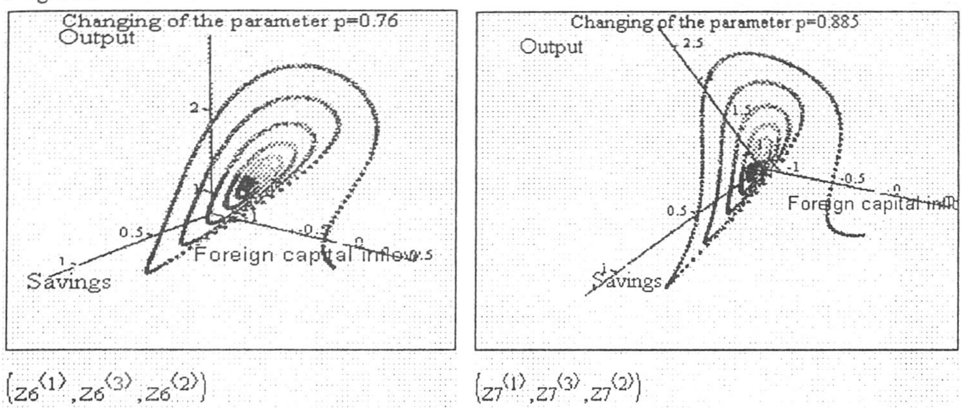


Figure 3

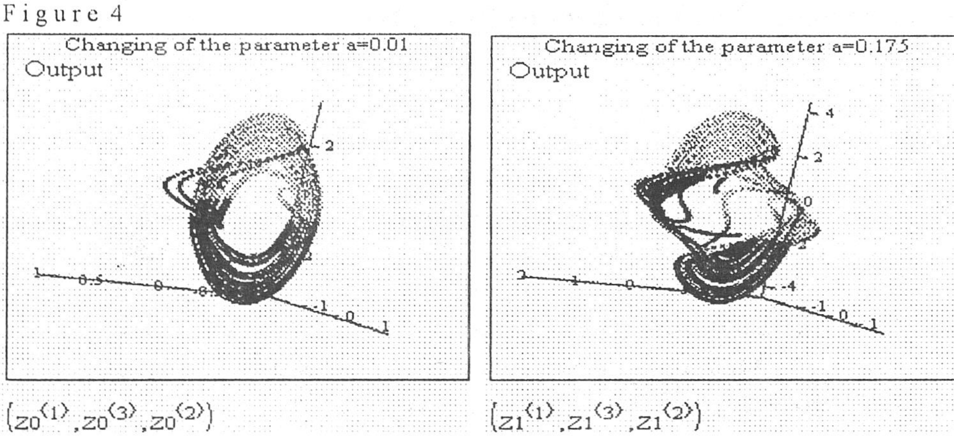


Example 2

Changing of the parameter a

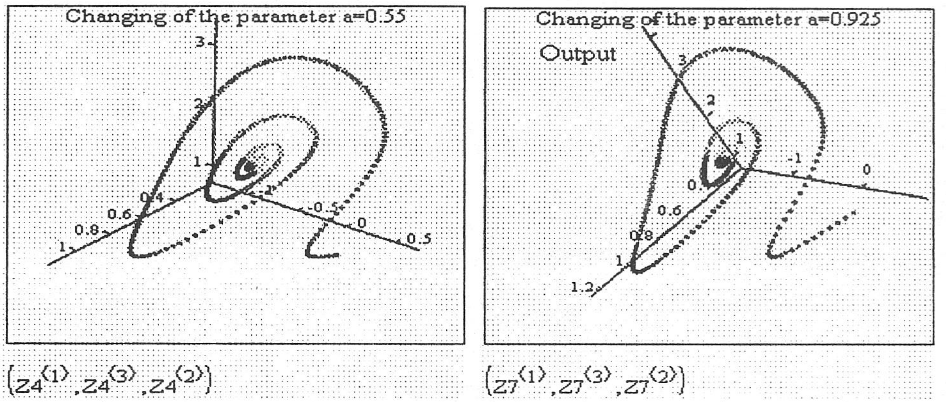
Let us consider the following values of parameters $a = 0.3$, $v = 1.25$, $p = 0.8$, $m = 0.8$, $r = 0.2$, $YP = 1$, and we solve the system of differential equations with the following initials $Y(0) = (0.45 \;\; 0.8 \;\; 0.035)^T$. The parameter a will be changed from the value 0.05 until 0.925 with other parameters unchanged. For low values of a this system is in an unstable state. For increasing values of a this system is in a stable state. Therefore this system under changing values of the parameter a has the sub critical Hopf bifurcation again. It means that when the parameter a is increasing from $a = 0.05$ till 0.925, the single fixed point changes its stability which is monitored in figures 4, and 5. We will now change piecemeal values of the parameters a in the following way $a = 0.05, 0.175, 0.55, 0.925$

Figure 4



In figure 4 it is possible to see the trajectories for the values of the parameter a for 0.01–0.175. In figure 5 are shown the situations for the values of the parameter a for 0.55–0.925.

Figure 5

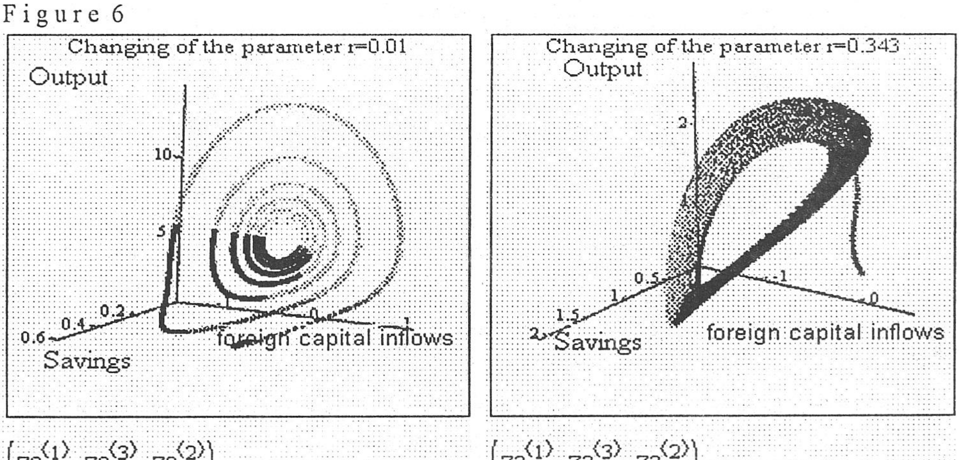


Example 3

Changing of the parameter r

Let us consider the following values of parameters $a = 0.3$, $v = 1.25$, $p = 0.8$, $m = 0.8$, $r = 0.2$, $YP = 1$, and we solve the system of differential equation with the following initials $Y(0) = (0.45 \;\; 0.8 \;\; 0.35)^T$. The parameter r will be changed from the value 0.007 until 0.343 and next from 0.455 till 0.791 with other parameters unchanged. For values of r less than 0.35 this system is *in a stable state* by the stability condition for $r < a$. For values of r larger than 0.35 this system is *in an unstable state*. Therefore this system under changing values of the parameter r has the so-called supercritical Hopf bifurcation. It means that when the parameter r is increasing from $r < 0.35$ to $r > 0.35$, the single fixed point changes its stability. We will now change piecemeal values of the parameters r in the following way $r = 0.01, 0.343, 0.455, 0.791$. In figure 6 it is possible to see the trajectories for the values of the parameter r for 0.01–0.343.

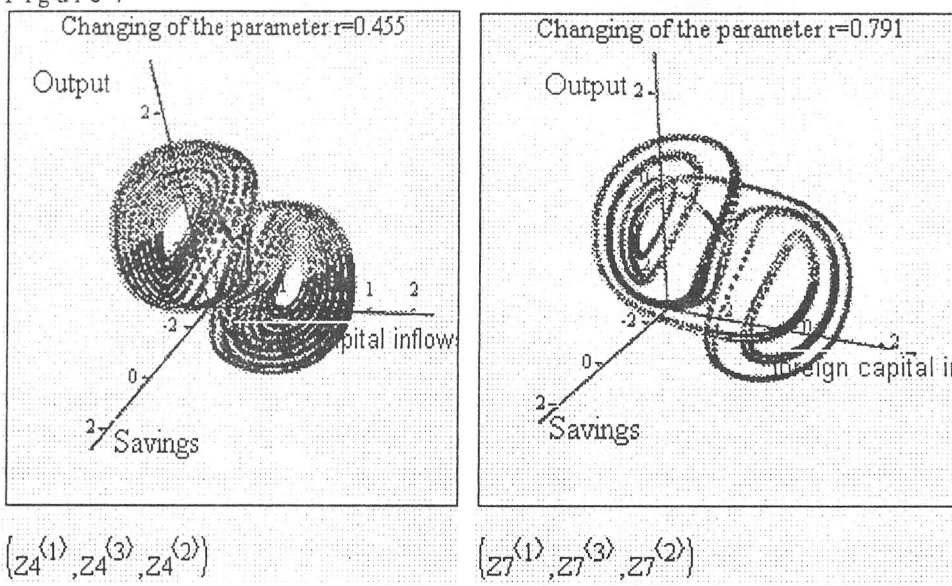
Figure 6



In figure 7 is shown the situation for the values of the parameter r for 0.455–0.791.

In figure 6 it is possible to see the trajectories for the values of the parameter r for 0.007 and for 0.343. In figure 7 is shown the situation for the values 0.455 and for 0.791.

Figure 7



Example 4

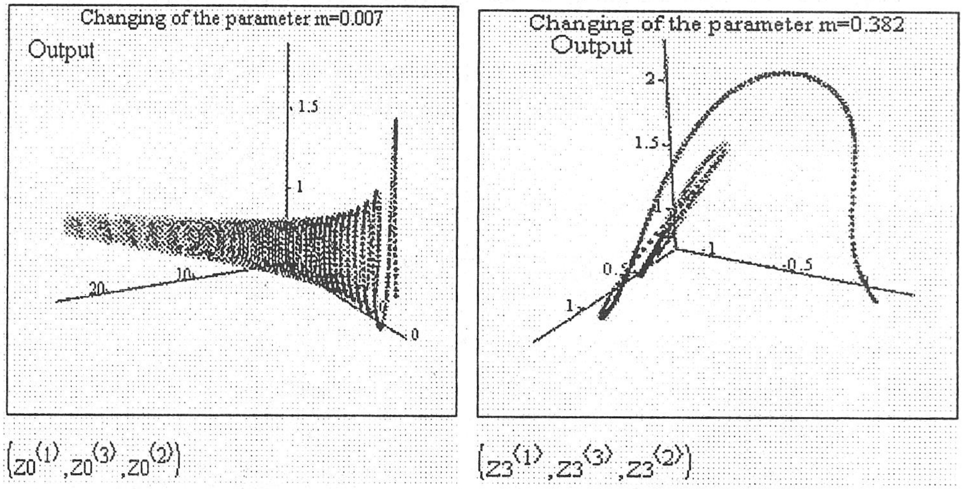
Changing of the parameter m

Let us consider the following values of parameters $a = 0.3$, $v = 1.25$, $p = 0.8$, $m = 0.8$, $r = 0.2$, $a = 0.05$, $v = 1.25$, $p = 0.8$, $m = 0.1$, $r = 0.04$, $YP = 1$, and we solve the system of differential equation with the following initials $Y(0) = 0.45\ 0.8\ \ 0.35)^T$. The parameter m will be changed from the value 0.007 until 0.382 and next from 0.507 till 0.882 with other parameters unchanged. For values of m less than 0.507 this system is in a stable state. For values of m larger than 0.507 this system is in an unstable state.

Therefore this system under changing values of the parameter m has the supercritical Hopf bifurcation again.

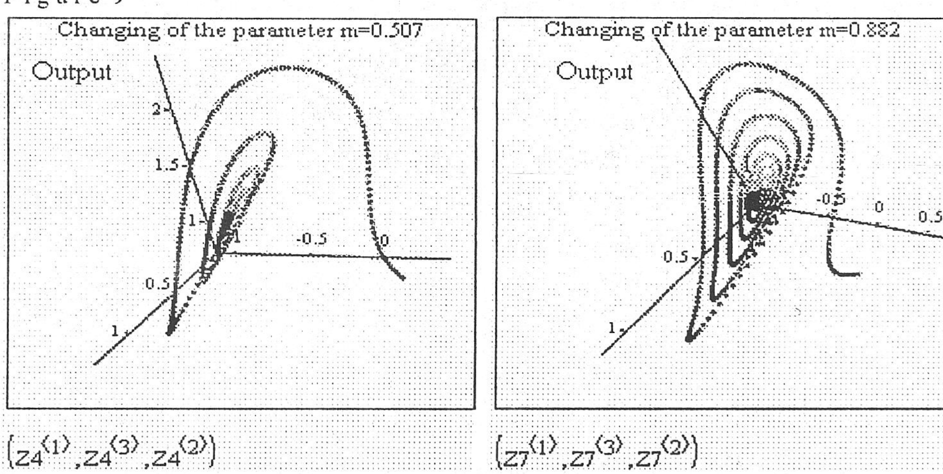
This means that when the parameter m is increasing from $m < 0.507$ to $m > 0.507$, the single fixed point changes its stability. We will now change piecemeal values of the parameters m in the following way $m = 0.007, 0.382, 0.507, 0.882$.

Figure 8



In figure 8 it is possible to see the trajectories for the values of the parameter m for 0.007–0.382. In figure 9 is shown the situation for the values of the parameter m for 0.507–0.882.

Figure 9



3. Conclusions

• An increasing of the capitalization of the profits, p, demonstrates well-known results in economics that the capitalization of profits causes the stabilization of the economic system. Our approach makes a possible to show a relation between a speed of convergence to stabilization and a level of capitalized profits.

• The parameter a, a variation of the marginal propensity to savings, has a similar behaviour as the parameter p. It is well-known result, that savings mean an economic stability

• If the debt refund/output ratio, r, is less than the variation of the marginal propensity to savings, a, then the system is in a stable state. If the debt refund/output ratio is greater than the variation of the marginal propensity to savings then this system converges to a limit cycle position.

• If the capital inflow/savings ratio, m, is less than double the ratio of capitalized profit p then the economic system is in a stable state. If the capital inflow/savings ratio is greater than double the ratio of capitalized profit then the system converges to the limit cycle. For higher difference between the capital inflows/savings ratio and double the ratio of capitalized profit this system converges to a chaotic state.

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Appendix A

Estimate a sign of the discriminant

$$\Delta = \left(\frac{g^3}{27} - \frac{g \cdot h}{6} + \frac{q}{2}\right)^2 + \left(\frac{h}{3} - \frac{g^2}{9}\right)^3 \tag{A1}$$

where

$$g = \frac{a \cdot m}{r} \qquad q = 2 \cdot \frac{m}{v} \cdot \left(a + \frac{p \cdot r}{m}\right) \qquad h = a + \frac{r}{v} \cdot \left(1 + 2 \cdot \frac{p}{m}\right)$$

By algebraic operations on the expression (A1) we get the following expressions

$$\left(\frac{g}{27} + \frac{q}{2} - g \cdot \frac{h}{6}\right)^2 = \frac{g^2}{729} + \frac{g \cdot q}{27} - \frac{g^2 \cdot h}{81} + \frac{q^2}{4} - \frac{g \cdot q \cdot h}{6} + \frac{g^2 \cdot h^2}{36} \tag{A2}$$

and

$$\left(\frac{h}{3} - \frac{g^2}{9}\right)^3 = \frac{h^3}{27} - \frac{g^2 \cdot h^2}{27} + \frac{h \cdot g^4}{81} - \frac{g^6}{729} \tag{A3}$$

Next algebraic operations on the expression (A2) we get the following expressions

$$\frac{g^2}{729} = \frac{a^2}{729} \cdot \frac{m^2}{r^2} \qquad \frac{g \cdot q}{27} = -\frac{2a^2}{27} \cdot \frac{m^2}{r \cdot v} - \frac{2a}{27} \cdot \frac{m}{v} \cdot p$$

$$-\frac{g^2 \cdot h}{81} = -\frac{a^3}{81} \cdot \frac{m^2}{r^2} - \frac{a^2}{81} \cdot \frac{m^2}{r \cdot v} - \frac{2a^2}{81} \cdot \frac{m \cdot p}{r \cdot v}$$

$$\frac{q^2}{4} = \frac{m^2}{v^2} \cdot a^2 + 2\frac{a \cdot m \cdot p \cdot r}{v^2} + \frac{p^2 \cdot r^2}{v^2}$$

$$-\frac{g \cdot h \cdot q}{6} = \frac{a^3}{3} \cdot \frac{m^2}{r \cdot v} + \frac{a^2}{3} \cdot \frac{m^2}{v^2} + \frac{2a^2}{3} \cdot \frac{m \cdot p}{v^2} + \frac{a^2}{3} \cdot \frac{m}{v} \cdot p + \frac{a \cdot p \cdot r}{3} \cdot \frac{m}{v^2} + \frac{2a \cdot r}{3} \cdot \frac{p^2}{v^2}$$

After the algebraic operations on the expression (A3) we get the following expressions

$$\frac{h^3}{27} = \frac{a^3}{27} + \frac{a^3}{9} \cdot \frac{r}{v} + \frac{2a^2}{9} \cdot \frac{r \cdot p}{m \cdot v} + \frac{a}{9} \cdot \frac{r^2}{v^2} + \frac{4a \cdot r^2 \cdot p}{9v^2 \cdot m} + \frac{4a \cdot r^2 \cdot p^2}{9v^2 \cdot m^2} + \frac{r^3}{27v^3} +$$

$$+ \frac{2}{9}\frac{r^3}{v^3} \cdot \frac{p}{m} + \frac{4r^3 \cdot p^2}{9v^3 \cdot m^2} + \frac{8}{27} \cdot \frac{r^3 \cdot p^3}{v^3 \cdot m^3}$$

$$\frac{g^4 \cdot h}{81} = \frac{a^5}{81} \cdot \frac{m^4}{r^4} + \frac{a^4}{81} \cdot \frac{m^4}{r^3 \cdot v} + \frac{2a^4}{81} \cdot \frac{m^3 \cdot p}{r^3 \cdot v} \qquad -\frac{g^6}{729} = -\frac{a^6}{729} \cdot \frac{m^6}{r^6}$$

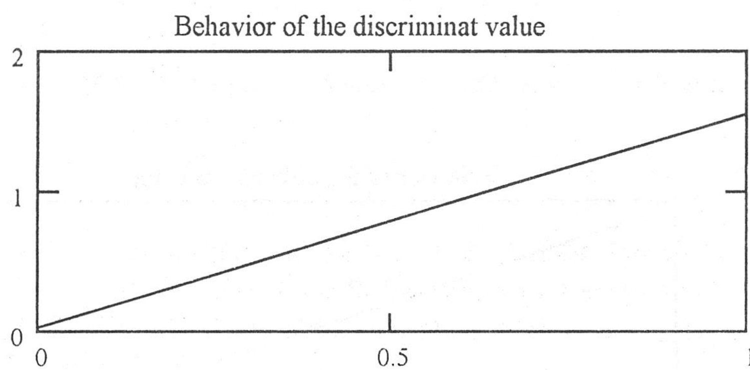
$$-\frac{g^2 \cdot h^2}{27} = -\frac{a^4}{27} \cdot \frac{m^2}{r^2} - \frac{2a^3}{27} \cdot \frac{m^4}{r \cdot v} - \frac{4a^3}{27} \cdot \frac{m \cdot p}{r \cdot v} - \frac{a^2}{27} \cdot \frac{m^2}{v^2} - \frac{4a^2 \cdot p}{27} \cdot \frac{m}{v^2} - \frac{4a^2}{27} \cdot \frac{p^2}{v^2}$$

A behaviour of the discriminant value for typical range of our system parameter values *a, m, p, v, r* are introduced in the following figures. Results have shown that the value of our discriminant is positive for all considered values of our parameters.

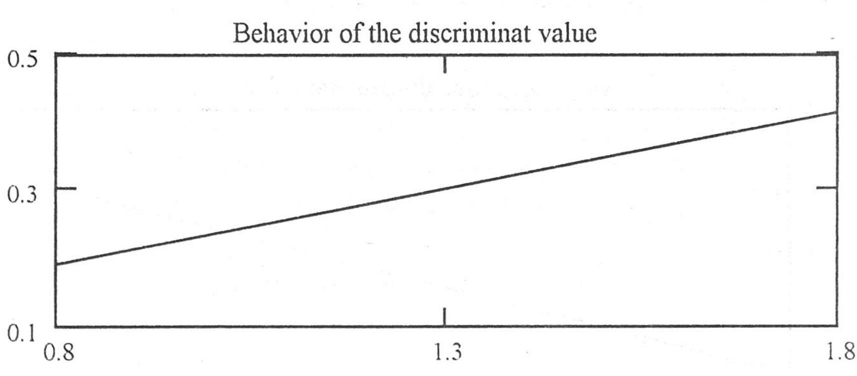
$a = 0.3$ $p = 0.8$ $v = 1.25$ $m = 0.8$ $r = 0.2$

Figures A1–A5

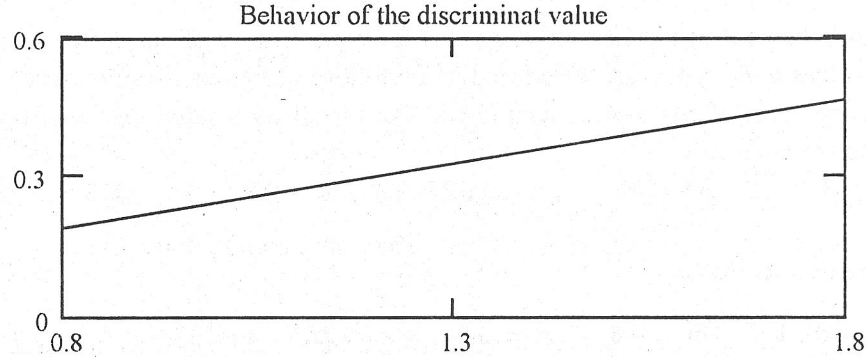
a := 0.. 1 m := 0.8 p := 0.8 v := 1.25 r := 0.2



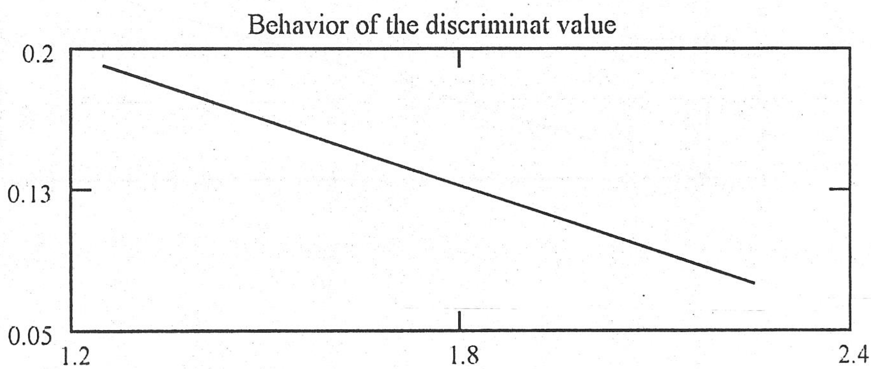
a := 0.3 m := 0.8.. 1.8 p := 0.8 v := 1.25 r := 0.2



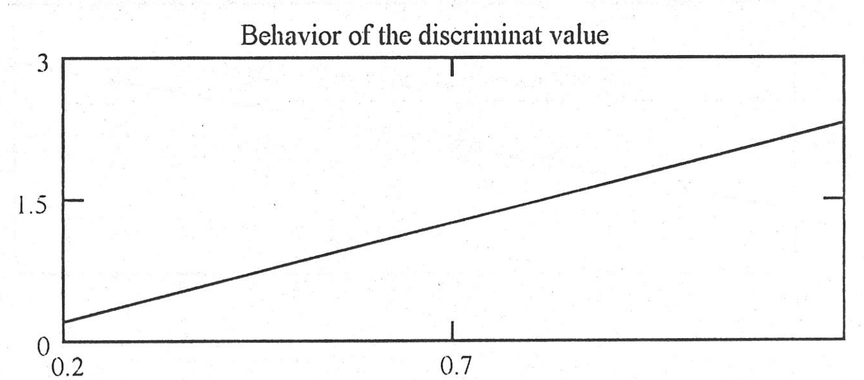
a := 0.3 m := 0.8 p := 0.8 .. 2 v := 1.25 r := 0.2



a := 0.3 m := 0.8 p := 0.8 v := 1.25 .. 2.25 r := 0.2



a := 0.3 m := 0.8 p := 0.8 v := 1.25 r := 0.2 .. 2



BIFURKAČNÍ CHOVÁNÍ EKONOMIE ZPŮSOBENÉ ZAHRANIČNÍMI INVESTICEMI

Miloslav S. VOŠVRDA

Původní motivací pro napsání tohoto článku byly výsledky získané z bifurkační analýzy makroekonomického modelu. Bylo překvapující, že drobné změny v parametrech dovedou způsobit nečekaně silné změny v chování makroekonomických indikátorů. Pro presentaci těchto výsledků byl vybrán model složený ze tří diferenciálních rovnic. První diferenciální rovnice popisuje chování úspor, druhá diferenciální rovnice popisuje vliv úspor a zahraničních investic na přírůstek výstupu systému. Třetí diferenciální rovnice popisuje chování zahraničních kapitálových trhů. Analýza se provádí tak, že je nalezena stacionární rovnováha, pokud existuje, a formulovány nutné a postačující podmínky pro stabilitu rovnováhy systému.

Pomocí Routh-Hurwitzova teorému jsou odvozeny nerovnosti mezi parametry pro splnění nutných a postačujících podmínek. Na příkladech je pak demonstrováno chování rovnovážných stavů ekonomických systémů, když dochází ke změně parametru.

Příklad první analyzuje chování systému v případě, že se mění hodnota reprezentující výši kapitalizovaného zisku. Analýza ukazuje, že s růstem hodnoty parametru p systém konverguje do stabilního stavu.

Druhý příklad analyzuje chování systému v případě, že se mění hodnota parametru, který reprezentuje rozsah mezního sklonu k úsporám. Ukazuje se, že pokud je sklon k úsporám příliš jednotvárně strukturován, vede to k cyklickému chování. Zvyšujeme-li hodnotu tohoto parametru, tzn. že se rozšiřují možnosti či struktura alokací úspor, vede to k stabilnímu chování ekonomického systému.

Příklad třetí, reprezentující formu splácení dluhu z výstupu systému, ukazuje na to, že pokud splácení představuje malou část výstupu systému, pak systém je v stabilním cyklickém stavu. Zvyšuje-li se podíl dluhu na výstupu systému, nad hladinu mezního sklonu k úsporám, pak systém konverguje k multicyklickým až chaotickým stavům.

Příklad čtvrtý ukazuje chování systému za předpokladu, že se mění hladina přílivu kapitálu k úrovni úspor. Je-li příliv kapitálu vzhledem k úsporám nízký, pak systém je ve stabilním stavu. Zvyšuje-li se příliv kapitálu k hladině úspor, dochází k rozkmitání systému.

Závěry z této analýzy jsou následující:

- zvyšování kapitalizace zisku vede ke stabilizaci ekonomického systému;
- rozšíření možností alokace úspor vede ke stabilitě;
- je-li podíl dluhu k outputu nižší než rozsah mezního sklonu k úsporám, pak je systém ve stabilním stavu;

- jestliže hladina přílivu kapitálu k úrovni úspor je menší než úroveň daná dvojnásobkem kapitalizovaného zisku, pak je systém ve stabilním stavu. Jestliže je tato hladina překročena, pak systém konverguje k limitním cyklům. Jestliže je tato hladina výrazně překročena, pak systém konverguje k chaotickému stavu.