OLG model technical paper (draft)*

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March 30, 2026

Abstract

In this paper ....

Keywords: OLG model, pension system ...

JEL Codes: E60....

*Domonkos acknowledges that this paper is the partial result of the project APVV under Grant No APVV-17-0551
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1 Introduction

2 Model formulation

An individual reaching 20 in t maximizes an intertemporal utility function of the form

$$u^t = \sum_{j=1}^{4} \beta^{j-1} \left(\ln c_j^t + \gamma_j \frac{(l_j^t)^{1-\theta}}{1-\theta} \right) + \gamma_{d_1} \ln d_1^t + \gamma_{d_2} \ln d_2^t \tag{1}$$

Table 1 shows the life-cycle of an individual reaching age 20 in t.

Variables (10 in total) to optimize:

- Savings: a_{1-3}
- Working time: n_{1-2}, $\tilde{n}_3$
- Early retirement: R
- Education: e_1
- Number of children d_{1-2}

Leisure time

$$\begin{aligned} l_1^t &= 1 - n_1^t - e_1^t - s_1 d_1^t && (2)\\ l_2^t &= 1 - n_2^t - s_2 d_1^t - s_1 d_2^t && (3)\\ l_3^t &= \Gamma \left[\pi \left((R^t(1-\tilde{n}_3^t)) \right)^{1-\frac{1}{\rho}} + (1-\pi)(1-R^t)^{1-\frac{1}{\rho}} \right]^{\frac{\rho}{\rho-1}} - s_2 d_2^t && (4)\\ l_4^t &= 1 && (5) \end{aligned}$$

Other variables

- Consumption: c_{1-4}
- Real wage per unit of effective labor: w
- Effective human capital: h_{1-3}
- Lump sum transfer that the government pays out to all individuals: z
- Consumption and labor tax rates: τ_c, τ_w

Age	20-35	35-50	50-65	65-80
Period	t	$t+1$	$t+2$	$t+3$
Work	n_1^t	n_2^t	$n_3^t = R^t \tilde{n}_3^t$	0
Study	e_1^t	0	0	0
Children	$s_1 d_1^t$	$s_2 d_1^t + s_1 d_2^t$	$s_2 d_2^t$	0
Leisure	$1 - n_1^t - e_1^t - s_1 d_1^t$	$1 - n_2^t - s_2 d_1^t - s_1 d_2^t$	$R^t(1-\tilde{n}_3^t) + (1-R^t) - s_2 d_2^t$	1

Table 1: Life-cycle of an individual of generation t

- Non-employment benefit replacement rates: b
- Maternity benefit replacement rate: b_m
- Time cost per child: s_{1-2}
- Financial cost to bring up offspring as a fraction of the after-tax wage income: ω_{1-2}
- Real interest rate: r

2.1 Budget constraints

$$\begin{aligned}(1+\tau_c)c_1^t + a_1^t + \omega_1 d_1^t w_t h_1^t n_1^t (1-\tau_w) = \\ w_t h_1^t n_1^t (1-\tau_w) + b w_t h_1^t (1 - n_1^t - e^t - d_1^t s_1) \\ + b_m w_t h_1^t d_1^t s_1 + z_t\end{aligned} \tag{6}$$

$$\begin{aligned}(1+\tau_c)c_2^t + a_2^t + (\omega_2 d_1^t + \omega_1 d_2^t) w_{t+1} h_2^t n_2^t (1-\tau_w) = \\ w_{t+1} h_2^t n_2^t (1-\tau_w) + b w_{t+1} h_2^t (1 - n_2^t - d_1^t s_2 - d_2^t s_1) \\ + (1 + r_{t+1}) a_1^t + b_m w_{t+1} h_2^t (d_1^t s_2 + d_2^t s_1) + z_{t+1}\end{aligned} \tag{7}$$

$$\begin{aligned}(1+\tau_c)c_3^t + a_3^t + \omega_2 d_2^t w_{t+2} h_3^t R^t \tilde{n}_3^t (1-\tau_w) = \\ w_{t+2} h_3^t R^t \tilde{n}_3^t (1-\tau_w) + b w_{t+2} h_3^t (R^t (1 - \tilde{n}_3^t) - d_2^t s_2) \\ + (1 + r_{t+2}) a_2^t + erb_3^t + b_m w_{t+2} h_3^t d_2^t s_2 + z_{t+2}\end{aligned} \tag{8}$$

$$(1+\tau_c)c_4^t = (1 + r_{t+3}) a_3^t + pb_4^t + z_{t+3} \tag{9}$$

2.2 Pension benefit

Basic first-pillar benefit

$$b1p_4^t = \rho_w \frac{1}{3} \sum_{j=1}^{3} w_{t+3} h_j^{t+4-j} n_j^t \tag{10}$$

$\rho_w = 50\%$ - reduction according to the second pillar

Parental pension

$$pp_4^t = \rho_P \sum_{j=1}^{2} w_{t+3} h_{4-j}^{t+j} n_{4-j}^{t+j} d_j^t \tag{11}$$

$\rho_P = 3\%$ according to the current setting

Maternity leave pension

$$mp_4^t = \rho_m \frac{1}{3} \sum_{j=1}^{2} w_{t+3} d_j^t s_1 h_j^{t+4-j} \tag{12}$$

$\rho_m = 0.6 \cdot \rho_w$ according to current setting

Second-pillar pension

$$b2p_4^t = \sum_{j=1}^{3} \left(\prod_{i=j}^{3} (1 + r_{t+i}) \right) w_{t+j-1} h_j^t n_j^t \tau_{2p} \tag{13}$$

- Reduce ρ_w according to the second pillar
- Include second-pillar contributions in τ_w
- Total τ_w = tax 19% + 13.4% + 36.2% - health insurance 15%=53.6%

Early retirement benefit

$$erb_3^t = (b1p_4^t + mp_4^t)(1 - \varepsilon(1 - R^t))(1 - R^t) + pp_4^t(1 - R^t) \tag{14}$$

- $(1 - R^t)$: duration of early retirement
- $\varepsilon(1 - R^t)$: pension reduction due to early retirement
- Reduction in Slovakia: 6% per year. Therefore, $\varepsilon = 0.06 \times 16 = 0.96$

Full pension

$$pb_4^t = (b1p_4^t + mp_4^t)(1 - \varepsilon(1 - R^t)) + pp_4^t + b2p_4^t \tag{15}$$

2.3 First order conditions

Substituting (2-9) into (1) and maximizing with respect to a_{1-3}, n_{1-2}, $\tilde{n}_3$, R, e_1, d_{1-2}, yields ten first order conditions.

Variables a_{1-3}

$$\frac{c_{j+1}^t}{c_j^t} = \beta(1 + r_{t+j})\ \forall j = 1, 2, 3 \tag{16}$$

Variables n_{1-2}, $\tilde{n}_3$

$$\frac{\gamma_1}{(l_1^t)^\theta} = \frac{1}{c_1^t}\frac{\partial c_1^t}{\partial n_1^t} + \beta^2 \frac{1}{c_3^t}\frac{\partial c_3^t}{\partial n_1^t} + \beta^3 \frac{1}{c_4^t}\frac{\partial c_4^t}{\partial n_1^t} \tag{17}$$

where

$$\frac{\partial c_1^t}{\partial n_1^t} = \frac{w_t h_1^t}{1 + \tau_c}((1 - \omega_1 d_1^t)(1 - \tau_w) - b)$$

$$\frac{\partial c_3^t}{\partial n_1^t} = \frac{\rho_w w_{t+3} h_1^{t+3}(1 - \varepsilon(1 - R^t))(1 - R^t)}{3(1 + \tau_c)}$$

$$\frac{\partial c_4^t}{\partial n_1^t} = \frac{1}{1 + \tau_c}\left(\frac{1}{3}\rho_w w_{t+3} h_1^{t+3}(1 - \varepsilon(1 - R^t)) + \tau_{2p} w_t h_1^t \prod_{i=1}^{3}(1 + r_{t+i})\right)$$

$$\frac{\gamma_2}{(l_2^t)^\theta} = \frac{1}{c_2^t}\frac{\partial c_2^t}{\partial n_2^t} + \beta \frac{1}{c_3^t}\frac{\partial c_3^t}{\partial n_2^t} + \beta^2 \frac{1}{c_4^t}\frac{\partial c_4^t}{\partial n_2^t} \tag{18}$$

where

$$\frac{\partial c_2^t}{\partial n_2^t} = \frac{w_{t+1} h_2^t}{1 + \tau_c}((1 - \omega_2 d_1^t - \omega_1 d_2^t)(1 - \tau_w) - b)$$

$$\frac{\partial c_3^t}{\partial n_2^t} = \frac{1}{3(1 + \tau_c)}\rho_w w_{t+3} h_2^{t+2}(1 - \varepsilon(1 - R^t))(1 - R^t)$$

$$\frac{\partial c_4^t}{\partial n_2^t} = \frac{1}{1 + \tau_c}\left(\frac{1}{3}\rho_w w_{t+3} h_2^{t+2}(1 - \varepsilon(1 - R^t)) + w_{t+1} h_2^t \tau_{2p} \prod_{i=2}^{3}(1 + r_{t+i})\right)$$

$$\frac{1}{c_3^t}\frac{\partial c_3^t}{\partial \tilde{n}_3^t} + \beta\frac{1}{c_4^t}\frac{\partial c_4^t}{\partial \tilde{n}_3^t} + \frac{\gamma_3}{(l_3^t)^\theta}\frac{\partial l_3^t}{\partial \tilde{n}_3^t} = 0 \tag{19}$$

where

$$\begin{aligned}
\frac{\partial c_3^t}{\partial \tilde{n}_3^t} &= \frac{R^t}{1+\tau_c}[((1-\omega_2 d_2^t)(1-\tau_w)-b)w_{t+2}h_3^t \\
&\quad +\frac{1}{3}\rho_w w_{t+3}h_3^{t+1}(1-\varepsilon(1-R^t))(1-R^t)] \\
\frac{\partial c_4^t}{\partial \tilde{n}_3^t} &= (\frac{1}{3}\rho_w w_{t+3}h_3^{t+1}R^t(1-\varepsilon(1-R^t)) + (1+r_{t+3})w_{t+2}h_3^t R^t \tau_{2p})/(1+\tau_c) \\
\frac{\partial l_3^t}{\partial \tilde{n}_3^t} &= -\Gamma\left[\pi\left((R^t(1-\tilde{n}_3^t)\right)^{1-\frac{1}{\rho}} + (1-\pi)(1-R^t)^{1-\frac{1}{\rho}}\right]^{\frac{1}{\rho-1}}\pi(1-\tilde{n}_3^t)^{-\frac{1}{\rho}}(R^t)^{1-\frac{1}{\rho}}
\end{aligned}$$

Variable R

$$\frac{1}{c_3^t}\frac{\partial c_3^t}{\partial R^t} + \beta\frac{1}{c_4^t}\frac{\partial c_4^t}{\partial R^t} + \frac{\gamma_3}{(l_3^t)^\theta}\frac{\partial l_3^t}{\partial R^t} = 0 \tag{20}$$

where

$$\begin{aligned}
\frac{\partial c_3^t}{\partial R^t} &= \frac{w_{t+2}h_3^t(\tilde{n}_3^t(1-\omega_2 d_2^t)(1-\tau_w) + b(1-\tilde{n}_3^t)) + \frac{\partial erb_3^t}{\partial R^t}}{1+\tau_c} \\
\frac{\partial erb_3^t}{\partial R^t} &= \frac{1}{3}\rho_w w_{t+3}h_3^{t+1}\tilde{n}_3^t(1-\varepsilon(1-R^t))(1-R^t) \\
&+ (b1p_4^t + mp_4^t)(-2\varepsilon R^t + 2\varepsilon - 1) - pp_4^t \\
\frac{\partial c_4^t}{\partial R^t} &= \frac{1}{3(1+\tau_c)}\rho_w w_{t+3}h_3^{t+1}\tilde{n}_3^t(1-\varepsilon(1-R^t)) + \varepsilon(b1p_4^t + mp_4^t)/(1+\tau_c) \\
&+ w_{t+2}h_3^t\tilde{n}_3^t\tau_{2p}(1+r_{t+3})/(1+\tau_c) \\
\frac{\partial l_3^t}{\partial R^t} &= \Gamma\left[\pi\left(R^t(1-\tilde{n}_3^t)\right)^{1-\frac{1}{\rho}} + (1-\pi)(1-R^t)^{1-\frac{1}{\rho}}\right]^{\frac{1}{\rho-1}} \times \\
&\quad \times\left(\pi\left(R^t(1-\tilde{n}_3^t)\right)^{-\frac{1}{\rho}}(1-\tilde{n}_3^t) - (1-\pi)(1-R^t)^{-\frac{1}{\rho}}\right)
\end{aligned}$$

Variable e

$$\frac{1}{c_1^t}\frac{\partial c_1^t}{\partial e^t} + \beta\frac{1}{c_2^t}\frac{\partial c_2^t}{\partial e^t} + \beta^2\frac{1}{c_3^t}\frac{\partial c_3^t}{\partial e^t} + \beta^3\frac{1}{c_4^t}\frac{\partial c_4^t}{\partial e^t} - \frac{\gamma_1}{(l_1^t)^\theta} = 0 \tag{21}$$

where

$$
\begin{aligned}
\frac{\partial c_1^t}{\partial e^t} &= \frac{-b w_t h_1^t}{1+\tau_c} \\
\frac{\partial c_2^t}{\partial e^t} &= \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} w_{t+1} h_1^t \frac{(1-\omega_2 d_1^t - \omega_1 d_2^t) n_2^t (1-\tau_w)}{1+\tau_c} + \\
&+ \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} w_{t+1} h_1^t \frac{b(1-n_2^t - d_1^t s_2 - d_2^t s_1) + b_m (d_1^t s_2 + d_2^t s_1)}{1+\tau_c} \\
\frac{\partial c_3^t}{\partial e^t} &= \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} w_{t+2} h_1^t \frac{(1-\omega_2 d_2^t) R^t \tilde{n}_3^t (1-\tau_w)}{1+\tau_c} + \\
&+ \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} w_{t+2} h_1^t \frac{b(R^t(1-\tilde{n}_3^t) - d_2^t s_2) + b_m d_2^t s_2}{1+\tau_c} \\
&+ \frac{\partial erb_t}{\partial e} / (1+\tau_c) \\
\frac{\partial erb_t}{\partial e} &= \frac{1}{3} \rho_w w_{t+3} h_1^t \cdot \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} \cdot x_{t+1} (x_{t+2}(n_1^t + n_2^t) + R^t \tilde{n}_3^t) \times \\
&\times (1-\varepsilon(1-R^t))(1-R^t) \\
&+ \frac{\rho_m}{3} w_{t+3} h_1^t \cdot \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} \cdot x_{t+1} x_{t+2} (s_1 d_1^t + s_1 d_2^t) \times \\
&\times (1-\varepsilon(1-R^t))(1-R^t) \\
&+ \rho_P w_{t+3} h_1^t \cdot \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} \cdot x_{t+1} (x_{t+2} n_2^{t+2} d_2^t + n_3^{t+1} d_1^t)(1-R^t) \\
\frac{\partial c_4^t}{\partial e^t} &= \frac{1}{3} \rho_w w_{t+3} h_1^t \cdot \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} \cdot x_{t+1} (x_{t+2}(n_1^t + n_2^t) + R^t \tilde{n}_3^t) \times \\
&\times (1-\varepsilon(1-R^t))/(1+\tau_c) + \\
&+ \rho_P w_{t+3} h_1^t \cdot \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} \cdot x_{t+1} (x_{t+2} n_2^{t+2} d_2^t + n_3^{t+1} d_1^t)/(1+\tau_c) + \\
&+ \frac{\rho_m}{3} w_{t+3} h_1^t \cdot \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} \cdot x_{t+1} x_{t+2} (s_1 d_1^t + s_1 d_2^t) \times \\
&\times (1-\varepsilon(1-R^t))/(1+\tau_c) + \\
&+ \tau_{2p} h_1^t (1+r_{t+3})(1+r_{t+2}) \cdot \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} w_{t+1} n_2^t / (1+\tau_c) + \\
&+ \tau_{2p} h_1^t (1+r_{t+3}) \cdot \frac{\partial \psi(e^t, g_y, q)}{\partial e^t} w_{t+2} n_3^t / (1+\tau_c) \\
n_3^t &= R^t \tilde{n}_3^t \\
x_t &= 1 + \psi(e^t, g_y, q) \\
\psi(e, g_y, q) &= \phi q \left(\nu g_y^{1-1/\kappa} + (1-\nu) e^{1-1/\kappa} \right)^{\sigma\kappa/(\kappa-1)} \\
\frac{\partial \psi(e, g_y, q)}{\partial e} &= \frac{\sigma\kappa}{\kappa-1} \phi q \left(\nu g_y^{1-1/\kappa} + (1-\nu) e^{1-1/\kappa} \right)^{\frac{\sigma\kappa}{\kappa-1}-1} (1-\nu)(1-1/\kappa) e^{-\frac{1}{\kappa}}
\end{aligned}
$$

Variable d_1

$$
\frac{\gamma_{d_1}}{d_1^t} - s_1 \frac{\gamma_1}{(l_1^t)^\theta} - \beta s_2 \frac{\gamma_2}{(l_2^t)^\theta} + \frac{1}{c_1^t} \frac{\partial c_1^t}{\partial d_1^t} + \beta \frac{1}{c_2^t} \frac{\partial c_2^t}{\partial d_1^t} + \beta^2 \frac{1}{c_3^t} \frac{\partial c_3^t}{\partial d_1^t} + \beta^3 \frac{1}{c_4^t} \frac{\partial c_4^t}{\partial d_1^t} = 0 \tag{22}
$$

where

$$\begin{aligned}
\frac{\partial c_1^t}{\partial d_1^t} &= w_t h_1^t \frac{(b_m - b)s_1 - \omega_1 n_1^t(1-\tau_w)}{1+\tau_c} \\
\frac{\partial c_2^t}{\partial d_1^t} &= w_{t+1} h_2^t \frac{(b_m - b)s_2 - \omega_2 n_2^t(1-\tau_w)}{1+\tau_c} \\
\frac{\partial c_3^t}{\partial d_1^t} &= \frac{\rho_m}{3} w_{t+3} h_1^{t+3} s_1(1-\varepsilon(1-R^t))(1-R^t)/(1+\tau_c) + \rho_P w_{t+3} h_3^{t+1} n_3^{t+1}(1-R^t)/(1+\tau_c) \\
\frac{\partial c_4^t}{\partial d_1^t} &= \rho_P w_{t+3} h_3^{t+1} n_3^{t+1}/(1+\tau_c) + \frac{\rho_m}{3} w_{t+3} s_1 h_1^{t+3}(1-\varepsilon(1-R^t))/(1+\tau_c)
\end{aligned}$$

Variable d_2

$$\frac{\gamma_{d_2}}{d_2^t} - \beta s_1 \frac{\gamma_2}{(l_2^t)^\theta} - \beta^2 s_2 \frac{\gamma_3}{(l_3^t)^\theta} + \beta \frac{1}{c_2^t}\frac{\partial c_2^t}{\partial d_2^t} + \beta^2 \frac{1}{c_3^t}\frac{\partial c_3^t}{\partial d_2^t} + \beta^3 \frac{1}{c_4^t}\frac{\partial c_4^t}{\partial d_2^t} = 0 \tag{23}$$

where

$$\begin{aligned}
\frac{\partial c_2^t}{\partial d_2^t} &= w_{t+1} h_2^t \frac{(b_m - b)s_1 - \omega_1 n_2^t(1-\tau_w)}{1+\tau_c} \\
\frac{\partial c_3^t}{\partial d_2^t} &= w_{t+2} h_3^t \frac{(b_m - b)s_2 - \omega_2 R^t \tilde{n}_3^t(1-\tau_w)}{1+\tau_c} \\
&+ \frac{\rho_m}{3} w_{t+3} h_2^{t+2} s_1(1-\varepsilon(1-R^t))(1-R^t)/(1+\tau_c) + \rho_P w_{t+3} h_2^{t+2} n_2^{t+2}(1-R^t)/(1+\tau_c) \\
\frac{\partial c_4^t}{\partial d_2^t} &= \rho_P w_{t+3} h_2^{t+2} n_2^{t+2}/(1+\tau_c) + \frac{\rho_m}{3} w_{t+3} s_1 h_2^{t+2}(1-\varepsilon(1-R^t))/(1+\tau_c)
\end{aligned}$$

Demographics

N_0^t, N_1^t, N_2^t, N_3^t, N_4^t population t in periods $0, 1, \ldots 4$, period 0 is childhood

$$N_0^t = N_1^{t-1} d_1^{t-1} + N_2^{t-2} d_2^{t-2} \tag{24}$$
$$N_1^t = N_0^t(1 - m_1^t - e_1^t + i_1^t) \tag{25}$$
$$N_2^t = N_1^t(1 - m_2^t - e_2^t + i_2^t) \tag{26}$$
$$N_3^t = N_2^t(1 - m_3^t - e_3^t + i_3^t) \tag{27}$$
$$N_4^t = N_3^t(1 - m_4^t - e_4^t + i_4^t) \tag{28}$$

2.4 Firms optimization problem

Profit maximization problem of the firm:

$$\max \Pi = K^\alpha H^{(1-\alpha)} - wH - \left(\frac{r}{1-\tau_k} + \delta\right) K$$

FOC 1

$$\frac{\partial \Pi}{\partial H} = (1-\alpha)\frac{K^\alpha}{H^\alpha} - w = 0$$

FOC 2

$$\frac{\partial \Pi}{\partial K} = \alpha \frac{K^{\alpha-1}}{H^{\alpha-1}} - \left(\frac{r}{1-\tau_k} + \delta\right) = 0$$

From the above first-order conditions, we derive the following equations for wages and the interest rate as follows:

Wages:

$$\boxed{\frac{\partial \Pi}{\partial H} = (1-\alpha)\frac{K^{\alpha}}{H^{\alpha}} = w} \tag{29}$$

Interest rate (rental proice of capital):

$$\boxed{(\alpha \frac{K^{\alpha-1}}{H^{\alpha-1}} - \delta)(1-\tau_k) = r} \tag{30}$$

2.5 Human capital

The growth rate of the effective human capital is as follows:

- Efficiency parameter: ϕ
- PISA score (OECD): q
- Share parameter: ν
- Elasticity of substitution: κ
- Productive government expenditures including health care expenditures (age 0-17) as percentage of output: g_y
- Time in education: e
- Scale parameter: σ

$$\psi(e, g_y, q) = \phi q \left(\nu g_y^{1-\frac{1}{\kappa}} + (1-\nu)e^{1-\frac{1}{\kappa}}\right)^{\sigma \frac{\kappa}{\kappa-1}} \tag{31}$$

$$H_t = n_1^t h_1^t N_1^t + n_2^{t-1} h_2^{t-1} N_2^{t-1} + n_3^{t-2} h_3^{t-2} N_3^{t-2}$$

Since

$$h_1^t = h_2^{t-1}$$

$$h_3^t = h_2^t = (1+\psi)h_1^t$$

$$h_3^{t-2} = h_2^{t-2} = h_1^{t-1} = \frac{h_2^{t-1}}{x^{t-1}} = \frac{h_1^t}{x^{t-1}}$$

$$x^{t-1} = 1+\psi$$

Then

$$\boxed{H_t = \left(n_1^t N_1^t + n_2^{t-1} N_2^{t-1} + \frac{n_3^{t-2}}{x^{t-1}} N_3^{t-2}\right) h_1^t} \tag{32}$$

Variable ψ

The growth rate of the human capital ψ depends on three inputs.

2.6 Government

$$\Delta D = D_{t+1} - D_t$$

$$\boxed{\Delta D = (G_y^t + G_c^t + B_t + PP_t + MB_t + Z_t) - (T_n^t + T_k^t + T_c^t) + r_t D_t}$$

Expenditures:

Government productive expenditures

$$G_y^t = g_y Y_t$$

Government consumption expenditures

$$G_c^t = g_c Y_t$$

Government non-employment benefits

$$B_t = N_1^t b w_t h_1^t (1 - n_1^t - e^t - d_1^t s_1) + N_2^{t-1} b w_t h_2^{t-1} (1 - n_2^{t-1} - d_1^{t-1} s_2 - d_2^{t-1} s_1) +$$
$$+ N_3^{t-2} R^{t-2} (1 - \tilde{n}_3^{t-2} - d_2^{t-2} s_2) b w_t h_3^{t-2}$$

Government maternity benefits

$$MB_t = N_1^t b_m w_t h_1^t d_1^t s_1 + N_2^{t-1} b_m w_t h_2^{t-1} (d_1^{t-1} s_2 + d_2^{t-1} s_1) + N_3^{t-2} b_m w_t h_3^{t-2} d_2^{t-2} s_2$$

Pension transfers

$$PP_t = N_4^{t-3} (p b_4^{t-3} - b2 p_4^{t-3}) + N_3^{t-2} e r b_3^{t-2}$$

Lump sum transfers

$$Z_t = z_t (N_1^t + N_2^{t-1} + N_3^{t-2} + N_4^{t-3})$$

Revenues:

Labour tax

$$T_n^t = \sum_{j=1}^{3} n_j^{t+1-j} w_t h_j^{t+1-j} \tau_w N_j^{t+1-j}$$

Capital tax

$$T_k^t = \tau_k (\alpha Y_t - \delta_k K_t)$$

Consumption tax

$$T_c^t = \tau_c \sum_{j=1}^{4} c_j^{t+1-j} N_j^{t+1-j}$$

Government Debt Servicing Costs

2.7 Equilibrium conditions

Equilibrium condition for an open economy

$$Y_t = C_t + I_t + G_t + NX_t$$

The open economy with Current Account, excluding Net Transfers, thus:

$$CA_t = NX_t + r_t F_t$$

Gross National income is then defined as follows:

$$\boxed{Y_t + r_t F_t = C_t + I_t + G_t + CA_t}$$

Then:

$$Y_t + r_t F_t - C_t - G_t = I_t + CA_t$$

$$S_t = I_t + CA_t$$

While:

$$F_t = A_t - K_t - D_t$$

$$A_t = a_1 + a_2 + a_3$$

$$G_t = G_{ct} + G_{yt}$$

$$CA_t = F_{t+1} - F_t$$

$$K_{t+1} = (1 - \delta_k)K_t + I_t$$

$$I_t = \Delta K_{t+1} + \delta_k K_t$$

Equilibrium condition for a closed economy (ALTERNATIVE APPROACH)

$$Y_t = C_t + I_t + G_t$$

$$Y_t - C_t - G_t = S_t = I_t$$

$$S_t = I_t$$

$$K_{t+1} = (1 - \delta_k)K_t + I_t$$

$$I_t = \Delta K_{t+1} + \delta_k K_t$$

2.8 List of variables and parameters of the model

Endogenous variables

Exogenous variable

Parameters