SOME COMPARATIVE MACROECONOMIC ANALYSIS OF VISEGRAD COUNTRIES IN THE PERIOD 1997-2010 USING DIFFERENTIAL GEOMETRY APPROACH

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Some Comparative Macroeconomic Analysis of Visegrad Countries in the Period 1997-2010 using Differential Geometry Approach

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Abstract: Paper presents a comparative macroeconomic analysis based upon GDP, inflation, and stock exchange index time series of Visegrad countries within the period 1997Q1 till 2010Q4. These macroeconomic quantities define a state space. We use stock indices BUX, PX, SAX, and WIG20 data from Budapest, Prague, Bratislava, and Warsaw, in particular. The raw macroeconomic data are very different in magnitudes when recorded in their natural units. In order to get suitable data sets, these raw data are normalized by pre-selected values. Such values are relatively very general ones, and they may be defined according to various concepts. Since we want to investigate and compare relative macroeconomic trajectories of the V4 countries, we accept averaged values of GDP, inflation, and stock exchange index of particular country in the year 2005 as the selected nominal values. Time parametric representation of given data in 3-D state space enables to tackle the macroeconomic development as unique evolution curve for each country properly. Based upon differential geometry approach, the Frenet frames are constructed at selected set of discrete points along the country evolution curve of the corresponding country. Investigation of frame translations provides both incremental and cumulative curve lengths, whereas frame rotations may generate country traces on unit sphere, in particular. The lengths of country evolution curves as well as the angular cumulative distances of spherical country traces provide reasonable novel measures for macroeconomic comparative analysis. Numerical results together with corresponding macroeconomic interpretations of the quantities generated are discussed in detail, and computer implementation is presented, too.

Keywords: Macroeconomic analysis, GDP, stock exchange index, inflation, time series analysis

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1 Introduction

Macroeconomic analysis is a very attractive field of research. Sure, there is also a lot of books, articles, and other references devoted to the topic and based upon various techniques and methods. Some of them are focused on summarizing data and monitoring facts, other ones concern theoretical modeling and are based upon various theoretical paradigms and approaches, for example the New Keynesian framework of macroeconomic analyses has become almost the most popular one, see e.g. Skála 2010. On the contrary, there are also works and studies which try to apply new insights and different methods in this challenging research field . The submitted paper ought to belong to this category.

We are concerned with a comparative macroeconomic analysis based upon GDP, inflation and stock exchange index time series of Visegrad countries within the period 1997Q1 till 2010Q4. A particular analysis of an evolution of the monetary policy transmission mechanisms of V4 countries is presented in Batóková 2010. Some algorithmic and numerical details of differential geometry approach to presented macroeconomic analysis of V4 countries is in Lukáš 2011.

Reasoning about the length of investigated period was motivated with three intuitive aspects – first, to eliminate prospective transient development of economy after 1989, second, to include period of relative stable and progressive economic development, and third, to involve periods which were impacted by global financial crisis, too. Parametric representation of data in 3-D state space enables to tackle the macroeconomic developments as curves in the state space. We do believe that our non-traditional approach provides new insights to macroeconomic analysis and is able to build new characteristics.

2 Macroeconomic data processing

We have used the data collected and presented in Telínová 2011, which had been downloaded from Eurostat, OECD databases and public databases of Stock exchanges of individual V4 countries, i.e. Czech Republic, Hungary, Poland and Slovak Republic, in abbreviation CZ, HU, PL, and SK, in particular. We have focused our analysis upon three important macroeconomic quantities – GDP, inflation and national stock exchange index, and in total, we have got four sets of three time series with 56 entries each.

Let ${}_k\boldsymbol{x}$ denotes a 3-D vector time series for k-th country, k = 1,2,3,4, where the prefix k is introduced in order to identify the particular country, i.e. k=1 ~ CZ, k=2 ~ HU, k=3 ~ PL, k=4 ~ SK, as we have adopted a simple lexicographical ordering of countries, in general.

Hence, each ${}_k\boldsymbol{x}$ has three 1-D time series components denoted ${}_kx_n$, n =1,2,3, where n=1 ~ stock exchange index, n=2 ~ GDP, n=3 ~ inflation rate, respectively. Further, each of these 1-D time series has 52 entries, thus we need even more subtle notation to identify any particular entry in a proper way.

Let ${}_kx_{n,m}$, m =1,2,…,52 denote individual entries. However, for some purposes we need to identify a particular year and quarter, too. Hence, we adopt the following notation ${}_kx_{n,m(i,j)}$, where the index function $m(i,j)$ has two in integer arguments, and it is defined $m(i,j)$ = 4(i–1)+j, where i=1,2,..,13 denotes sequence of years 1997, 1998,.., 2010, and j=1,2,3,4 stands for quarters of year Q1,..,Q4.

Raw time series data sets downloaded for V4 countries are denoted ${}_k\boldsymbol{y}$ and their entries ${}_ky_{n,m}$, m=1,2,…,56, respectively. However, such data are given in their natural units, which are rather different in their numerical values. In order to make them suitable for our analysis based on differential geometry approach the raw data ${}_k\boldsymbol{y}$ are normalized, i.e. scaled by pre-selected nominal values, thus yielding the desired time series ${}_k\boldsymbol{x}$. The normalizing procedure is given by expressions (1) and (2).

$$
{}_k r_n = \sum_{j=1}^{4} {}_k y_{n,m(9,j)}\ , \quad k = 1,2,3,4, \text{ and } n = 1,2,3, \tag{1}
$$

where the index function $m(9,j)$, j =1,2,3,4, i = 9, issues sequence of values {33,34,35,36}.

Quantities ${}_k r_n$ represent the pre-selected scaling factors and give averaged values for corresponding V4 countries macroeconomic time series. In a role of reference year we have selected the year 2005, hence i=9, since we simply accepted this year as a break-even year between a ‘stable development’ period, and a period accused by global financial crisis. Of course, it could be a matter of discussion, but the scaling procedure is rather flexible and could be applied for any other selected basis year, a quarter, or another period averaged.

Vector time series ${}_k\boldsymbol{x}$ with their components are given by (2), where $^{\mathrm{T}}$ stands for vector transposition, as usual.

$$
\begin{gathered}
{}_k x_{n,m} = {}_k y_{n,m} / {}_k r_n, \quad m = 1,2,\ldots,56, \text{ and } k = 1,2,3,4, \ n = 1,2,3, \\
{}_k\boldsymbol{x} = ({}_k x_{1,m}, {}_k x_{2,m}, {}_k x_{3,m})^{\mathrm{T}}.
\end{gathered} \tag{2}
$$

3 Differential geometry approach and the results

We refer to Chern et al. 2000, and Gray et al. 2006, for more details on differential geometry as being our rather short and very individual reference selection list, only. In general, differential geometry deals with curves, surfaces or more general manifolds in analytic way.

In order to get a curve, our first step is to define interpolants of ${}_k\boldsymbol{x}$. First, we choose piecewise linear interpolation, and the second one is cubic spline interpolation, which both provide continuous functions.

Let ${}_k\boldsymbol{p}(t)$, ${}_k\boldsymbol{q}(t)$ denote piecewise linear and cubic spline interpolants of vector time series ${}_k\boldsymbol{x}$ defined on interval [1,56] given by (3a) and (3b), respectively.

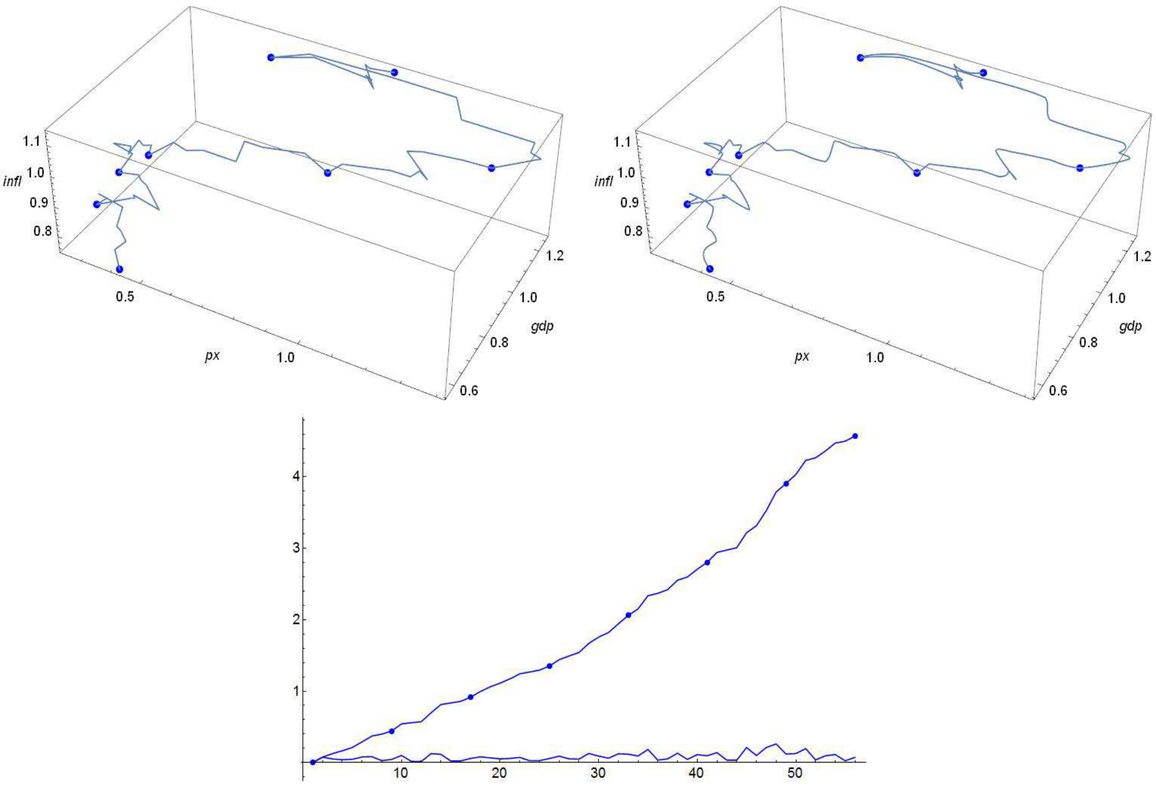
Let d be a time series with m entries, then $s_1(t;d)$ and $s_3(t;d)$ stand for scalar piecewise linear and cubic spline interpolants, respectively, defined on interval $[1,m]$, and built by well-known procedures. Hence, we may write the interpolants ${}_k\boldsymbol{p}(t)$, ${}_k\boldsymbol{q}(t)$, for k-th country, in the following way.

$$
{}_k\boldsymbol{p}(t) = (s_1(t;{}_k x_{1,m}), s_1(t;{}_k x_{2,m}), s_1(t;{}_k x_{3,m}))^{\mathrm{T}}, \tag{3a}
$$

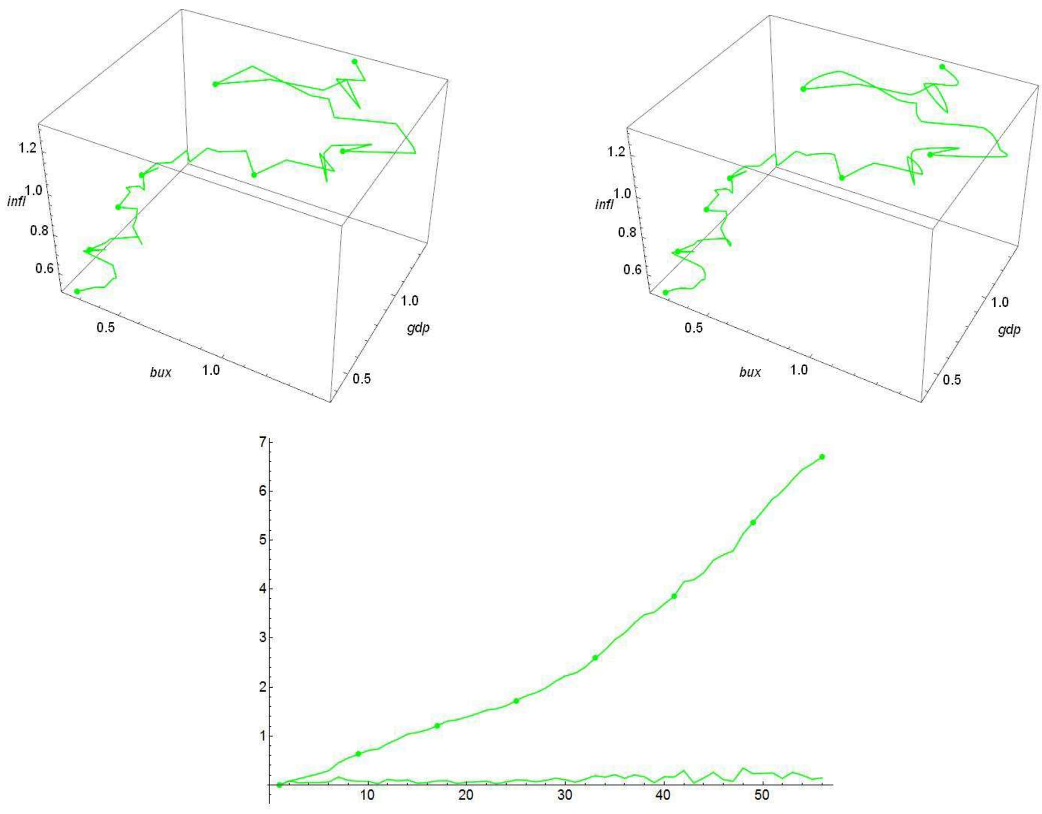
$$
{}_k\boldsymbol{q}(t) = (s_3(t;{}_k x_{1,m}), s_3(t;{}_k x_{2,m}), s_3(t;{}_k x_{3,m}))^{\mathrm{T}}. \tag{3b}
$$

.

The 3-D macroeconomic trajectories of corresponding V4 countries are presented in next four pictures, i.e. **Pic.1** for CZ, **Pic.2** for HU, **Pic.3** for PL, and **Pic.4** for SK, respectively. The axes are denoted *gdp*, *infl*, and *px*, *bux*, *vig20*, and *sax*, respectively.

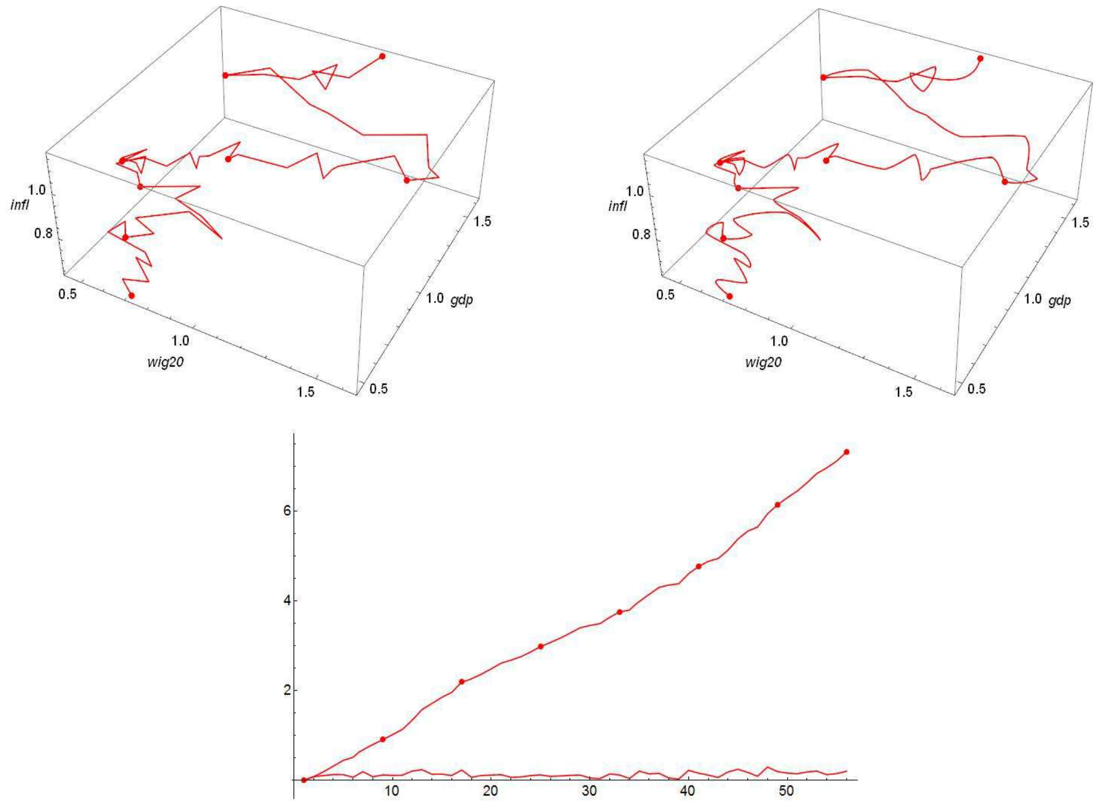


Pic.1: CZ - macroeconomic trajectory
Source: own calculation

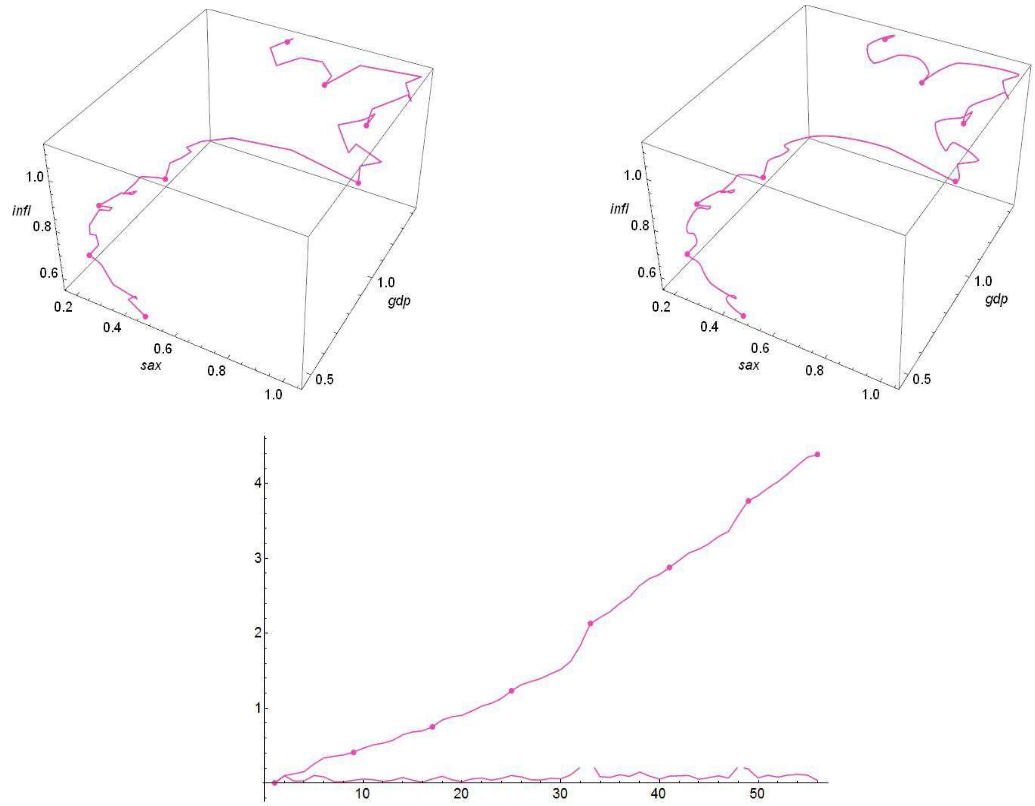


Pic.2: HU - macroeconomic trajectory

Source: own calculation

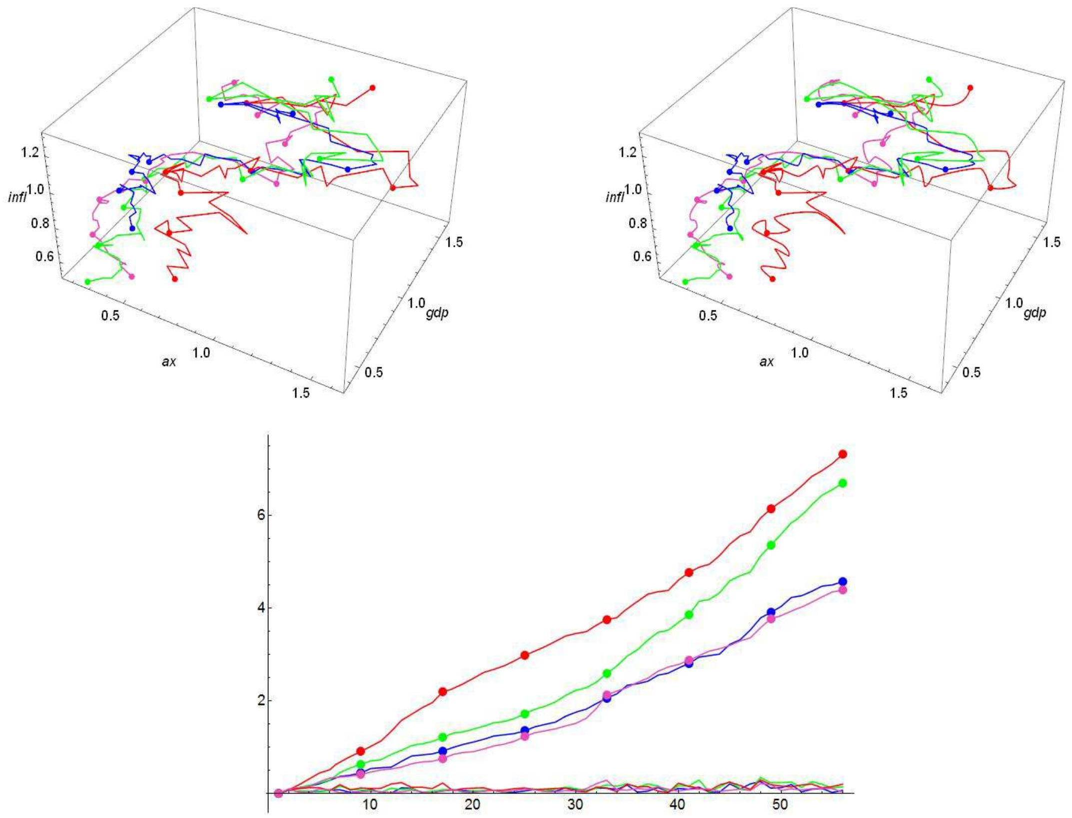


Pic.3: PL - macroeconomic trajectory
Source: own calculation

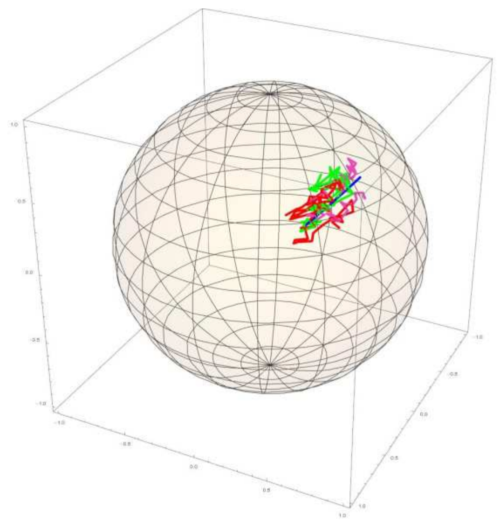


Pic.4: SK - macroeconomic trajectory
Source: own calculation

Systematically in all four pictures, i.e. **Pic.1 – Pic.4,** on the left side, there is plotted the piecewise linear interpolant of the country macroeconomic trajectory, whilst on the right side, there is cubic spline interpolant, and bellow is the corresponding length of this trajectory. The dots on the curves denote dates: 97Q1, 99Q1, 01Q1, 03Q1, 05Q1, 07Q1, 09Q1, and 10Q4, thereby giving better insight to country macroeconomic evolution sweeps along the whole time span [1997Q1, 2010Q4]. At the first glance inspecting these pictures, we may conclude that there is not a great difference between piecewise linear and cubic spline interpolants in all V4 countries, in general.



Pic.5: CZ, HU, PL, SK - macroeconomic trajectories
Source: own calculation



Pic.6: CZ, HU, PL, SK – angular traces on unit sphere
Source: own calculation

In **Pic.5** we are able to see all V4 country macroeconomic evolution curves. On the left, there are depicted their continuous piecewise linear interpolants ${}_k\boldsymbol{p}(t)$, k=1,2,3,4, while on the right, there are the cubic spline interpolants ${}_k\boldsymbol{q}(t)$. Bellow, we see cumulated lengths of the country macroeconomic evolution curves. The longest curve belongs to HU, the second one as to the length has PL, and finally, the CZ and SK curves are very similar. However, inspecting precisely the global lengths of both countries from 1997Q1 till 2010Q4, we are able to identify that the shortest path belongs to SK, while the CZ curve is a little bit longer.

Having for each k-th country vector functions ${}_k\boldsymbol{p}(t)$, ${}_k\boldsymbol{q}(t)$, which are at least continuous, we are able to construct well-known Frenet frame at any point t.

Let ${}_k\boldsymbol{F}(t)$ denotes the Frenet frame at point t along the k-th country curve, which defines corresponding local basis at this point being composed with three specific vectors called – tangent vector, normal vector, and binormal vector, respectively

$$ {}_k\boldsymbol{F}(t) = ({}_k\boldsymbol{\tau}(t), {}_k\boldsymbol{\upsilon}(t), {}_k\boldsymbol{\beta}(t))^{\mathrm{T}}, \; k = 1, .., 4, \tag{4} $$

where tangent vector ${}_k\boldsymbol{\tau}(t)$ is defined in different way depending upon algebraic degree of corresponding interpolant function as follows

- in case of continuous piecewise linear interpolant ${}_k\boldsymbol{p}(t)$, we get just finite collection of tangent vectors ${}_k\boldsymbol{\tau}(t_n) = {}_k\boldsymbol{\pi}_n = {}_k\boldsymbol{p}(t_{n+1}) - {}_k\boldsymbol{p}(t_n)$, at discrete set of time $\{t_n\}$, n = 1, .., 55, where t_1 = 1997Q1, and t_{55} = 2010Q3.

- in case of cubic spline interpolant, we have ${}_k\boldsymbol{\tau}(t) = d({}_k\boldsymbol{q}(t))/dt$, as usual.

Introducing moving Frenet frame along a curve, differential geometry provides definition of curve length in 3-D. In general, let $\boldsymbol{z}(t) = (z_1(t), z_2(t), z_3(t),)^{\mathrm{T}}$ be a smooth curve defined on closed interval $[a,b]$. Then, its length $l(a,b)$ is given as follows

$$ l(a,b) = \int_a^b \|\dot{\boldsymbol{z}}(t)\|\, dt, \qquad \dot{\boldsymbol{z}}(t) = (\dot{z}_1(t), \dot{z}_2(t), \dot{z}_3(t))^{\mathrm{T}}, \\ \dot{z}_i(t) = dz_i(t)/dt, \; i = 1,2,3. \tag{5} $$

However, the lengths of V4 country macroeconomic evolution curves presented in **Pic.5** are approximated by summations of corresponding line segments ${}_k\boldsymbol{\pi}_n$, actually. More precisely, the length ${}_kL(t_1,t_{56})$ of k-th country is defined in following way

$$ {}_kL(t_1,t_{56}) = \sum_{n=1}^{55} \|{}_k\boldsymbol{\pi}_n\|, \quad k = 1, .., 4, \tag{6} $$

where $\|\boldsymbol{w}\|$ denotes usual Euclidean norm of any vector $\boldsymbol{w}$.

Having calculated collections of tangent vectors for all V4 countries based upon ${}_k\boldsymbol{p}(t)$, we may get rather untraditional characteristics of country macroeconomic evolution curves - vortex motions, which are depicted in **Pic.6**.

First, we normalize the tangent vectors at their points t_n, and then we translate them parallel to the origin, i.e. t_1, where we also locate a centre of unit sphere S(**0**; 1) in 3-D. In particular, technical details are given by following expressions

$$ {}_k\boldsymbol{u}(t_n) = {}_k\boldsymbol{\pi}_n \,/\, \| {}_k\boldsymbol{\pi}_n \|, \quad k = 1, .., 4, \quad n = 1, .., 55, $$
$$ {}_k\boldsymbol{\omega}_n \text{ ... parallel translated vectors } {}_k\boldsymbol{u}(t_n) \text{ to the centre of unit sphere } S(\mathbf{0}; 1) \tag{7} $$

The traces of vortex motions of country macroeconomic evolution curves given in **Pic.6** are calculated as arcs on the S(**0**; 1) connecting apexes of ordered set of unit vectors ${}_k\boldsymbol{\omega}_n$ given by natural ordering $n = 1, .., 55$. The centre of S(**0**; 1) is $\mathbf{0} = (0,0,0)^T$.

4 Conclusion

We have analyzed some macroeconomic evolution curves of V4 countries and their lengths and vortex motions by differential geometry approach.

Since the presented approach and the whole framework of macroeconomic analysis is rather untraditional one, we can expect frequent discussion, interpretation and clarification of introduced quantities and presented results, as well. We may propose to use these quantities for a new quantitative measuring of macroeconomic frictions and inefficiencies, at least of some kinds of such phenomena. But the precise definition of them is still open.

However, we consider the analysis interesting and promising, and further research is ongoing.

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